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# LONGITUDINAL FEEDBACK DYNAMICS IN STORAGE RINGS WITH SMALL SYNCHROTRON TUNES *

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*Abstract*

We analyze the dynamics of multibunch longitudinal instabilities including bunch-by-bunch feedback under the assumption that the synchrotron tune is small. We find that increasing the feedback response does not always guarantee stability, even in the ideal case with no noise. As an example, we show that if the growth rate of a cavity-driven mode is of the order of the synchrotron frequency, then there are parameter regions for which the instability cannot be controlled by feedback irrespective of its gain. We verify these calculations with tracking simulations relevant to the APS-U, and find that the dynamics do not depend upon whether the longitudinal feedback relies on phase-sensing or energy-sensing technology. Hence, this choice should be dictated by measurement accuracy and noise considerations.

## THEORY

We will investigate multibunch stability in the presence of longitudinal feedback. For simplicity we assume that the ring is uniformly filled, and that all perturbing wakefields are approximately constant over each bunch. In this case the complex frequency $\Omega$ describing multibunch oscillations of mode $\mu$ satisfies the following dispersion relation:

$$\begin{aligned}1 = &\left[\frac{4i\pi I_{\rm tot}\sigma_t^2}{\alpha_c(\gamma mc^2/e)\sigma_\delta^2 T_0}\sum_{p=-\infty}^{\infty}\omega_{p,\mu}Z_\parallel(\omega_{p,\mu})\right.\\ &\left.-\frac{24\pi G\sigma_t^2}{\alpha_c^2\sigma_\delta^2 T_0^2}\sum_{p=0}^{\infty}\mathscr{C}_p e^{ip\Omega T_0}\right]\\ &\times\int d\mathcal{J}\;\bar{F}(\mathcal{J})\sum_{m=1}^{\infty}\frac{m^2\left|z_m(\mathcal{J})/\sigma_z\right|^2}{[\Omega/\omega(\mathcal{J})]^2-m^2}.\end{aligned} \tag{1}$$

Here, $I_{\rm tot}$ is the total current, $\alpha_c$ is the momentum compaction, $\sigma_z = c\sigma_t$ and $\sigma_\delta$ are rms bunch length and energy spread, respectively, $T_0 = 2\pi/\omega_0$ is the revolution period, $\bar{F}(\mathcal{J})$ and $z_m(\mathcal{J})$ are the equilibrium distribution function of action $\mathcal{J}$ and the $m^{\rm th}$ Fourier component of the longitudinal coordinate $z$, while for $M$ bunches $\omega_{p,\mu} = \omega_0(pM+\mu)+\Omega$. The first term in square brackets includes the long-range wakefields, while the second contains the feedback of gain $G$ and $\mathscr{C}_p$ that we describe shortly.

We will illustrate the basic stability properties contained in Eq. (1) by restricting ourselves to a specific form for the long-range wakefield. In particular, we consider longitudinal instabilities driven by a single higher-order mode (HOM) in the rf system, in which case the sum over the long-range impedance can be limited to the single term where the HOM resonance line most closely overlaps a revolution harmonic. Then, we can reduce the dispersion relation to

$$\begin{aligned}\frac{\alpha_c\sigma_\delta}{\sigma_t} = &\left[2\Lambda\frac{i+\varpi}{1+\varpi^2}+\frac{6\sigma_t G}{\alpha_c\sigma_\delta T_0^2}\sum_{p=0}^{\infty}\mathscr{C}_p e^{ip\Omega T_0}\right]\\ &\times\int d\mathcal{J}\;4\pi\bar{F}(\mathcal{J})\sum_{m=1}^{\infty}\frac{m^2\left|z_m(\mathcal{J})/\sigma_z\right|^2}{[\Omega/\omega(\mathcal{J})]^2-m^2},\end{aligned} \tag{2}$$

where we define the nominal growth rate $\Lambda$ and normalized detuning $\varpi$ in terms of the HOM shunt impedance $R_s$, quality factor $Q \gg 1$, and frequency $\omega_{\rm HOM}$ via

$$\Lambda = \frac{\sigma_t\omega_{\rm HOM}I_{\rm tot}R_s}{2\sigma_\delta(\gamma mc^2/e)T_0}, \quad \varpi = 2Q\frac{\omega_{\rm HOM}-p\omega_0-\Omega}{\omega_{\rm HOM}}. \tag{3}$$

We assume that the effect of the feedback is given by a finite impulse response (FIR) filter whose coefficients are determined by a least-square fit of the phase's derivative [1]:

$$\sum_{p=0}^{\infty}\mathscr{C}_p e^{ip\Omega T_0} = \sum_{p=0}^{N-1}\frac{(N-1)-2p}{N(N^2-1)}e^{ip\Omega T_0}. \tag{4}$$

We will furthermore assume that the feedback samples the longitudinal motion over a "short" number of turns, so that $NT_0$ is much smaller than both the synchrotron period and the growth time of the instability. Then, we expand the feedback sum to lowest order in $\Omega T_0$, in which case our results match those for both the usually employed sin-wave FIR filter [2] and for a simple energy-detection based system; this is because all these feedbacks contribute to the dispersion relation as the derivative of the bunch position $\propto -i\Omega T_0$.

Let's first assume that the longitudinal rf potential is quadratic in $z$. The resulting simple harmonic motion has $2\pi\sigma_\delta\sigma_z\bar{F}(\mathcal{J}) = e^{-\mathcal{J}/\sigma_z\sigma_\delta}$ and $z_{\pm1} = \sigma_z\sqrt{\mathcal{J}/2\sigma_z\sigma_\delta}$, so that the dispersion relation reduces to

$$\Omega^2-\omega_s^2 = 2\omega_s\Lambda\frac{i+\varpi}{1+\varpi^2}+\frac{6G}{T_0^2}\sum_{p=0}^{\infty}\mathscr{C}_p e^{ip\Omega T_0}. \tag{5}$$

Solutions to Eq. (5) give the complex frequencies $\Omega$ for a storage ring in which longitudinal feedback works to stabilize multibunch oscillations driven by a single higher-order cavity mode. While it is straightforward to solve numerically, we will make a few additional simplifications.

First, we assume that the width of the HOM resonance is much broader than the complex mode frequency, $\omega_{\rm HOM}/2Q \gg |\Omega|$. The APS-U cavities have HOMs whose widths are typically a few kHz, while $|\Omega|/2\pi \sim \omega_s/2\pi \lesssim$ 400 Hz. This implies that we can neglect the contribution of $\Omega$ to the detuning $\varpi$. Next, we assume that the FIR feedback filter retains a small number of turns such that both

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$\omega_s T_0$ and $|\Omega T_0|$ are much less than $1/N$. Under these two assumptions we obtain a quadratic equation with solutions

$$\Omega_\pm = -\frac{iG}{2T_0} \pm \left[\omega_s^2 - \frac{G^2}{4T_0^2} + 2i\omega_s\Lambda\frac{1-i\varpi}{1+\varpi^2}\right]^{1/2}. \quad (6)$$

Instability is indicated when $\Im(\Omega) > 0$. We will compare the general solution to simulations shortly, but before that we consider two simple limits.[1]

In the first limit, we assume that the synchrotron frequency is much larger than either the damping provided by the feedback or the growth from the instability. Mathematically this amounts to expanding the square root in (6) assuming that $\omega_s \gg G/T_0$ and $\omega_s \gg \Lambda$, which gives

$$\Omega_\pm \approx \pm\omega_s\left(1 + \frac{\Lambda}{\omega_s}\frac{\varpi}{1+\varpi^2}\right) - i\left(\frac{G}{2T_0} - \frac{\Lambda}{1+\varpi^2}\right). \quad (7)$$

This limit corresponds to an oscillator that is weakly perturbed by the feedback and the HOM, and is common in the literature. The mode oscillates with a frequency that is shifted from $\omega_s$ by the small amount $\propto \Lambda/\omega_s$. In addition, the feedback results in an effective damping rate $G/2T_0$, so that the mode is damped or grows depending upon whether $G/2T_0$ is larger or smaller than the instability growth rate $\Lambda/(1+\varpi^2)$.

The other limit we want to consider is when the feedback gain is chosen to be as large as possible to maximally damp any instability. In this idealization we take $G \gg \omega_s T_0$ and $G \gg |\Omega|T_0$, and find that one mode is stable and decays as $e^{-i\Omega\_s/c} \approx e^{-(G/T_0)s/c}$, while the other, potentially unstable mode has the complex frequency

$$\Omega_+ \approx \frac{2\omega_s\Lambda T_0}{G(1+\varpi^2)} - \frac{i\omega_s}{G}\left(\omega_s T_0 + \frac{2\Lambda T_0\varpi}{1+\varpi^2}\right). \quad (8)$$

Hence, when the HOM frequency is just below a revolution harmonic, $\varpi < 0$, we find that an instability exists when

$$-\frac{2\Lambda\varpi}{1+\varpi^2} > \omega_s \quad (9)$$

regardless of the feedback gain $G$. The left-hand side is maximized when $\varpi = -1$ for which $\Lambda > \omega_s$ implies instability. Furthermore, if the nominal growth rate $\Lambda \gtrsim 2\omega_s$ the dynamics is unstable for a range of negative detunings, while stability reigns when $\varpi \geq 0$.

To summarize, we find that when the feedback and HOM wakefields are weak perturbations Eq. (7) implies stability is guaranteed if $G > \Lambda T_0/2$. On the other hand, the high gain limit leads to stable motion for positive detunings $\varpi \geq 0$, but an instability can arise for $\varpi < 0$ when $\Lambda > \omega_s$ regardless of the feedback gain.

## TRACKING SIMULATIONS

To verify our conclusions, we performed a number of multibunch `elegant` simulations [3, 4]. Our simulations tracked 48 bunches for 100K turns through the RC4 variant of the APS-U lattice, whose linear and lowest-order nonlinearities were modeled using the `ILMATRIX` element. Synchrotron radiation was applied once per turn using the `SREFFECTS` element, and an `RFMODE` element simulated one cavity HOM whose frequency is near 921 MHz and chosen to exite the $\mu = 29$ coupled-bunch mode with fixed $Q = 10.4 \times 10^4$ and variable $R_s$ and detuning $\varpi$. Longitudinal feedback was applied using paired `TFBPICKUP` and `TFBDRIVER` elements that transform the detected phase to an energy kick via a 10-term FIR filter whose coefficients are given in Eq. (4); we found nearly identical results if the `TFBPICKUP` averages the energy centroid over 10 turns. Finally, the RF cavity parameters were chosen to set $\sigma_t \approx 52$ ps, which is the same bunch length as the planned dual rf system tuned to flatten the potential.

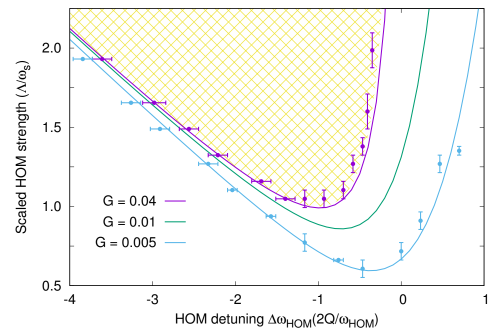


Figure 1: Regions of multibunch stability as a function of the HOM detuning $\varpi$. Theory predicts that the regions below the purple, green and blue lines are stable for feedback gains labelled. The points plot results from `elegant` simulations that assume $G = 0.04$ and $G = 0.005$; the top/right or bottom/left of the "error bars" indicate parameters where tracking displays unstable or stable motion, respectively. The yellow cross-hatched region is predicted to be unstable for any feedback gain $G < 1$.

We begin with an rf potential that is approximately quadratic, and the "correct" 52 ps bunch length is obtained by adopting a fictitious 39.1 MHz rf system with voltage $V = 3.78$ MV. This gives a synchrotron frequency $\omega_s/2\pi \approx 167$ Hz, and we expect that the "high gain limit" discussed previously applies when the feedback gain $G \gtrsim \omega_s T_0 \approx 4 \times 10^{-3}$. When the growth rate becomes large we found that the easiest way to identify an instability was by monitoring the HOM voltage for evidence of exponential growth.

We summarize our results in Fig. 1, where the endpoints of the error bars indicate the boundaries of stabilty as found in tracking. The theory Eq. (6) predicts that the regions below the purple, green and blue solid lines are stable for feedback gains of $G = 0.04$, 0.01, and 0.005, respectively. Furthermore, the yellow cross-hatched region is theoretically predicted to be unstable for any feedback gain $G < 1$.

[1] Synchrotron emission can be added to the harmonic potential results by replacing $G/T_0 \to G/T_0 + 2/\tau_z$ for longitudinal damping time $\tau_z$.

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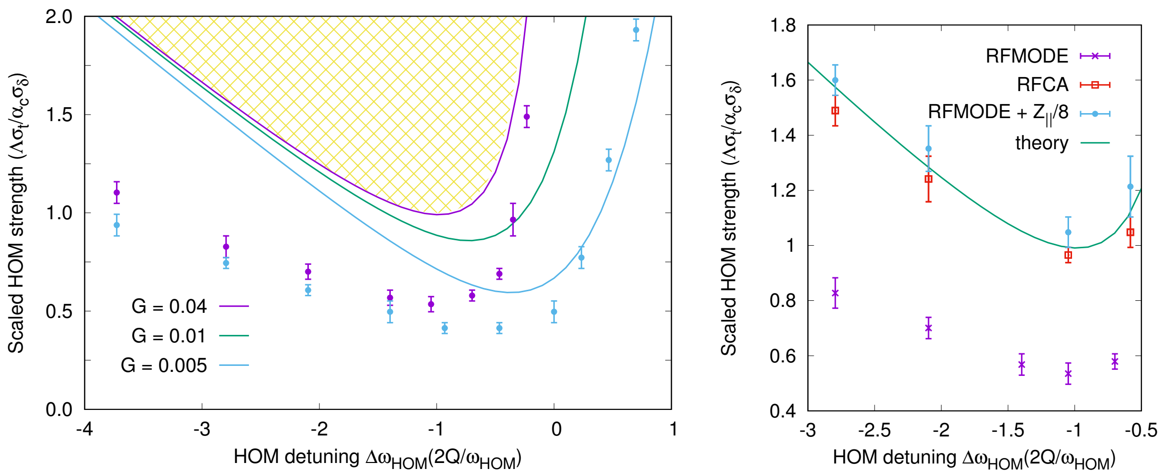


Figure 2: Regions of multibunch stability. The theory for a flattened potential is indistiguishable from that in Fig. 1, while the simulation points on the left plot simulate the APS-U's self-consistent double rf system with two RFMODE elements. The right panel plots results for a self-consistent double rf system with no impedance (purple) and with the ring $Z_\|/8$ (red), while the blue points employ prescribed rf fields modelled using RFCA elements.

For the next series of simulations we included the full dynamics of the APS-U's double rf system. Hence, we returned the main rf frequency to 352 MHz, and simulated the main cavities using 12 RFMODE elements that include beam loading and whose cavity voltage and phase were maintained with an rf feedback loop [5]. The bunch lengthening was provided by a passive, 4$^{\text{th}}$ harmonic RFMODE cavity that had $R_s = 61.8$ MΩ, $Q = 6 \times 10^5$, and $\Delta f = 14.5$ kHz.

We compare theory and tracking in Fig. 2. Here, the theory solved the full dispersion relation (1) assuming that the distribution function $\bar{F}(\mathcal{J})$ and the Fourier harmonics $z_m(\mathcal{J})$ were those of a quartic oscillator. The resulting stability predictions are practically identical to the findings for a simple harmonic oscillator shown before, so that the scaled Figures look identical if we divide the HOM strength by $\alpha_c\sigma_\delta/\sigma_t \sim 0.8\langle\omega_s\rangle$. We compare the theory to the simulation results on the left-hand side of Fig. 2. While the theory predicts essentially no change from the simple harmonic case, the simulations indicate that the maximum stable $\Lambda$ is reduced by about half.

Further investigations revealed that the simulation predictions depend upon the details of the longitudinal potential. We illustrate this the right-hand plot of Fig. 2, where we consider several scenarios but only for a few detuning values and only with the gain $G = 0.04$. The green line is the theory, but now the tracking uses one of the following:

**Purple** Self-consistent RFMODE elements (same as left).
**Red** Prescribed, flattened potential from 2 RFCA elements.
**Blue** Same as purple but with the ring impedance $Z_\|/8$.

Tracking in a prescribed, flattened longitudinal potential formed with two RFCA elements agrees quite well with the theory, verifies our calculation. Additionally, tracking indicates that the longitudinal impedance can increase stability by a factor of two or more: the stability is doubled when we add a ZLONGIT element with one-eighth of the longitudinal impedance, which approximately gives the bunch lengthening expected in 324 bunch mode; adding the impedance of $Z_\|/4$ increases stability even further (not shown). In any event, all cases display similar behavior, namely, that the strong feedback effectively damps collective motion when $\Delta\omega_{\text{HOM}} > 0$, but can only control instabilities whose strength $\Lambda \lesssim \alpha_c\sigma_\delta/\sigma_t$ when the detuning is small but negative. Nevertheless, it appears that detailed predictions require simulations that include all the details. This last finding is somewhat surprising, and we still do not fully understand why it should be true to the extent observed.

## CONCLUSIONS

We have used theory and tracking to show that there are cases when longitudinal feedback cannot damp instabilities, even in the ideal case with no noise. These results are particularly relevant to ultra-low emittance rings for which the instability growth rates can approach or even exceed the very small synchrotron frequency. Our results are largely independent of how the feedback is employed, so that the choice of feedback scheme should be dictated by noise levels and signal detection efficiency/accuracy.

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