# SCU ENDS CONFIGURED AS PHASE SHIFTER\*

M. Qian<sup>†</sup>, Argonne National Laboratory, Lemont, IL, USA

## Abstract

Phase shifters are usually needed in the interspace of a permanent magnet (PM)-based undulator array for purposes of phase matching when the field strength is changing. Unlike the PM-based undulators, the superconducting undulator (SCU) can change its end field with the help of varying currents in the end coils. By setting the end coil currents the phase-matching function could be realized, thus eliminating the need for standalone phase shifters, saving the interspace, as well as reducing the mechanical complexity of an undulator array. We developed a technique for determining the SCU end coil currents for phase shifting purposes and demonstrated it through numerical simulations. The procedure as well as the simulation results are described in this paper.

#### **INTRODUCTION**

In free electron laser (FEL) applications, undulators are installed in a line to form an undulator array. In the interspace between two undulator segments, there are elements such as phase shifter, quadrupole and steering magnets, and beam position monitor that take up longitudinal space. The packing fraction of an undulator array is defined as the ratio of the total length of the undulators to the total length of the whole array [1]. In most cases, a higher packing fraction is favored. To achieve a higher packing fraction without suppressing the interspace length, the hybrid permanent magnet undulators (HPMUs) for FELs can be up to 5 meters long. However, the lengths of the in-operating SCUs seldom exceed 1.5 m because of the engineering difficulties of fabricating a very long iron magnet core.

Given that the length of an individual SCU is limited, decreasing the interspace length should be considered. Unlike the HPMU cases, the end field of an SCU can be adjusted by independently powered coils. This opens the possibility of using the end field as a phase shifter. If the need for a standalone phase shifter can be eliminated, not only could the packing fraction be increased, but also the assembly of the SCU array would be significantly facilitated.

In this paper, we present our simulation experiments of configuring the adjustable SCU end coils as a phase shifter. We start with a technique to manipulate and optimize the trajectory in an SCU with the help of variable end coil currents, then we describe a feasible configuration using the end coils as phase shifters based on this technique.

### SIMULATION MODEL

Based on the symmetries of the magnetic field profile, an SCU can be categorized as an odd symmetrical configuration with even-numbered magnetic poles or an even symmetrical configuration with odd-numbered poles. In our study, a primary focus was given to the even symmetrical model.

The numerical experiments were carried out with the magneto-statics computer code OPERA3D [2], which is widely used in insertion device design and simulation. In the model environment the *z*-axis overlaps with the longitudinal direction of the SCU, x is the horizontal direction, and y is the main magnetic field direction.

The geometry of the SCU model is based on a previous device built at the APS [3]. The SCU has two soft iron cores; on each of them there are soft iron poles facing the gap. The coils are vertically wound on the cores. Figure 1 shows the 3D view of the OPERA model end part, while Table 1 presents the basic parameters of the model. Three end coils have adjustable currents; the ratios of these currents to the regular coil current are noted as  $c_1$ ,  $c_2$ , and  $c_3$ , respectively.



Figure 1: 3D view of the end part of the SCU OPERA model.

Table 1: Basic Parameters of the SCU Model

Parameter	Value	unit
Turns per coil	53	
Current	<588	А
Core and pole	1008 soft	
material	iron	
Period length	21	mm
Gap	8	mm
Pole $y^*z$	4.9*4.4	mm
Core $x^*y$	100*32	mm

A global magnetic field in the SCUs was observed that caused a trajectory distortion [4]. This distortion is scaled to the square of the undulator length and is insignificant when the model is short. Thus, we set our simulation models to have 52 poles and a total length of 0.5 m, which is long enough to capture the main trajectory distortion.

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In this section, we present the technique to manipulate and optimize the trajectory in the SCU. At this stage, we observe the trajectory segment of an SCU cell, which starts from the midpoint of one interspace and ends at the midpoint of the next interspace. The distance from the undulator core end to the center of the interspace is 20 cm in this study.

A linear fit was performed to the oscillating part of the trajectory first, then the fitted straight line was used as a reference and was removed from the trajectory. Three types of errors can be seen:

- Angle error at the end of the trajectory segment,  $E_{\alpha}$ .
- Shift error at the end of the trajectory segment,  $E_s$ .
- Trajectory distortion in the oscillating part of the main trajectory,  $E_m$ . This is caused by the magnetic flux line from the end traveling along the core deep into the longitudinal center of the SCU gap, creating a global magnetic field that adds a parabolic-shaped curvature to the trajectory.

These three types of trajectory errors are illustrated in Figure 2.



Figure 2: Unoptimized trajectory with three types of errors illustrated.

For an ideal trajectory, all three types of errors should be zeroed. Changing any of the coil currents will change the three trajectory errors. Suppose the end coils are powered by currents of  $c_{1,0}$ ,  $c_{2,0}$ , and  $c_{3,0}$ , and the end angle error, end shift error, and main trajectory distortion from the simulation are  $E_{\alpha,0}$ ,  $E_{s,0}$ , and  $E_{m,0}$  respectively. To zero these errors by adjusting the end coil currents is equivalent to solving the following equation system:

$$\boldsymbol{M} \cdot \begin{bmatrix} \delta \boldsymbol{c}_1 \\ \delta \boldsymbol{c}_2 \\ \delta \boldsymbol{c}_3 \end{bmatrix} + \begin{bmatrix} \boldsymbol{E}_{\alpha,0} \\ \boldsymbol{E}_{s,0} \\ \boldsymbol{E}_{m,0} \end{bmatrix} = \boldsymbol{0}. \tag{1}$$

The unknowns  $\delta c_1$ ,  $\delta c_2$ , and  $\delta c_3$  are the adjustments of the three end coil currents. **M** is a  $3 \times 3$  matrix whose elements are the partial derivatives that describe the effects of the end coil currents on the three errors. For instance, the element in the first row, second column is:

$$M_{1,2} = \frac{\partial E_{\alpha}}{\partial c_2} |_{c_{1,0}, c_{2,0}, c_{3,0}}.$$
 (2)

The elements in the matrix M can be obtained by tweaking the end currents by a small fraction in the simulation model and observing the resultant changes in the three types of trajectory errors. Due to the evident nonlinear nature of the simulation model-these partial derivatives in **M** varying with changing end coil currents and the above equation system being merely a linearized version with the higher order partial derivatives neglected—the obtained  $\delta c_n$  would not work. It is impractical to obtain the information about the higher order derivatives; instead, an iterative method must be used to solve Eq. (1). The end coil current adjustments in each iteration were scaled in the manner shown in Eq. (3) to make sure the maximum of their absolute values does not exceed the limit before they were applied to the model,

$$\begin{bmatrix} \delta c_1 \\ \delta c_2 \\ \delta c_3 \end{bmatrix} = \begin{cases} \begin{bmatrix} \delta c_1 \\ \delta c_2 \\ \delta c_3 \end{bmatrix}, & \max(|\delta c_n|) \le \sigma \\ \frac{\sigma}{\max(|\delta c_n|)} \begin{bmatrix} \delta c_1 \\ \delta c_2 \\ \delta c_3 \end{bmatrix}, & \max(|\delta c_n|) > \sigma \end{cases}$$
(3)

where  $n = \{1, 2, 3\}$ , and  $\sigma$  is the maximum allowed adjustment. In our study, we set  $\sigma = 0.015$  to guarantee fast convergence. The iteration stops when the absolute values of obtained adjustments are smaller than a threshold, which is 1E-4 in our study. After that, the final result was obtained.

Figure 3 shows the optimized trajectory at 588 A current. The corresponding end coil currents are  $\{c_1, c_2, c_3\} =$ {0.0357, 0.3656, 0.824}. The peak field at this regular coil current is 1.62 T.



Figure 3: Optimized trajectory with all three types trajectory errors corrected.

#### PHASE SHIFTER CONFIGURATION

When assembling the trajectory segments of identical SCUs into an undulator array trajectory, the main trajectory distortion has to be zeroed. On the other hand, it is not required to have both the end angle and end shift zeroed to satisfy the periodicity conditions of the array. To make sure the trajectory segment repeats itself, the trajectory at the two ends of an SCU has to satisfy:

$$\begin{cases} E_{s}|_{z-} = E_{s}|_{z+} \\ E_{\alpha}|_{z-} = E_{\alpha}|_{z+}, \end{cases}$$
(4)

where z- and z+ denote the two ends of the trajectory segment of an SCU. An SCU of even symmetrical configuration always has  $E_{\alpha}|_{z-} = -E_{\alpha}|_{z+}$ , so the second requirement becomes  $E_{\alpha}|_{z-} = E_{\alpha}|_{z+} = 0$ . While the first requirement is automatically satisfied due to the symmetry,

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the  $E_s$  term could be removed from Eq. (1) when optimizing the SCU trajectory in a periodic array; meanwhile one of the unknowns has to be removed to become a free parameter. We use c3 as a scanning (free) parameter, while c1 and c2 are optimized to zero the end angle and trajectory distortion. Figure 4 shows two optimized trajectories of c3=0.815 and c3=0.895, respectively.



Figure 4: Two optimized trajectories at a regular coil current of 588 A with different values for  $c_3$ .

The chicane-like trajectory at the ends, in the case of c3=0.895, cost the electron more time to pass the interspace. Analysis of phase slippages at the midpoint of the interspace referenced to the main trajectory shows the relative phase difference between these two trajectories (phase shift) is 270°. The total phase shift in the interspace between these two cases is 540°, significantly larger than the one-wave length requirement of  $2\pi$  for phase shifters, indicating that there are two c3s between 0.815 and 0.895 that satisfy the phase matching condition. The midpoint is where the quadrupole is to be installed, and the beam trajectory should pass through the center of the quadrupole. Therefore, for the c3=0.895 case, the main part of the trajectories has to be offset by -3.2 µm in the x-direction, which is supposed to have no significant impact on the FEL performance.

More simulations were performed to demonstrate the behavior at various  $c_{35}$  and two regular coil currents of 588 A and 200 A. The peak field at 200A is 0.84 T. The phase shifts  $c_{15}$  and  $c_{25}$  as functions of  $c_{25}$  are shown in Figures 5, 6 and 7.



Figure 5: Phase shifts as functions of  $c_3$  at regular coil currents 588 A and 200 A.



Figure 6:  $c_1$  as functions of  $c_3$  at regular coil currents 588 A and 200 A.



Figure 7:  $c_2$  as functions of  $c_3$  at regular coil currents 588 A and 200 A.

For both currents, the phase shifts cover sufficient ranges for phase shifters. The end coil current tuning ranges are acceptable. The smoothness of the curves will facilitate the implementation of this method because interpolations on these curves are allowed.

#### CONCLUSION

We developed a technique to optimize and steer the simulated end trajectory of an SCU in an FEL array by varying end coil currents and demonstrated that using this technique, the phase slippage between two neighboring SCUs could be shifted by sufficient amplitudes. Thus the independently powered SCU end coils can work as integrated phase shifters.

#### REFERENCES

- D. Nguyen, "Superconducting undulators and cryomodules for X-ray Free-Electron Lasers," presented at the 2022 North American Partical Accelerator Conf. (NAPAC'22), Albuquerque, NM, USA, Aug. 2022, paper THYE3, this conference.
- [2] https://www.3ds.com/productsservices/simulia/ products/opera/
- [3] Y. Ivanyushenkov et al., "Development and operating experience of a short-period superconducting undulator at the Advanced Photon Source," Phys. Rev. ST Accel. Beams 18, 040703 (2015). doi:10.1103/PhysRevSTAB.18.040703
- [4] D. Arbelaez and S. Prestemon, "Correction schemes for superconducting undulators", *Superconducting Undulators* for Advanced Light Sources Virtual Workshop, Apr. 2021, https://indico.desy.de/event/28501/contributi ons/99142/attachments/65373/80753/Arbelaez\_20 210421\_SCU\_Workshop.pdf

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