ISBN: 978-3-95450-232-5 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2022-WEPA63

EXTENSIONS OF THE COMPLEX (IQ) BASEBAND RF CAVITY MODEL INCLUDING RF SOURCE AND BEAM INTERACTIONS*

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Abstract

This paper extends prior work describing a complex envelope (i.e., baseband) dynamic model of excited accelerator RF cavities, including the effects of frequency detuning, beam loading, reflections, multiple drive ports and parasitic modes. This model is presented here in closed-form transfer function and state-variable realizations, which may be more appropriate for analytic purposes. Several example simulations illustrate the detailed insight into RF system behavior afforded by this model.

TRANSFER FUNCTION MODEL

Stated below are the baseband Laplace in-rail (cosine) and cross-rail (sine) impulse transimpedance responses of an intrinsic cavity, as developed in [1]. Here, the polynomial coefficients have been recast to conform to standard control theory nomenclature.

$$Z_c(s) = \frac{b_{1c}s + b_{2c}}{s^2 + a_1 s + a_2};$$
$$Z_s(s) = \frac{b_{1s}s + b_{2s}}{s^2 + a_1 s + a_2};$$

where:

$$a_1 = \frac{2}{\tau};$$
$$a_2 = \left(\frac{1}{\tau^2} + \Delta\omega^2\right);$$
$$b_{1c} = \frac{R_c}{\tau};$$
$$b_{2c} = \left(\frac{R_c}{\tau}\right) * \left(\frac{1}{\tau} - \frac{\Delta\omega}{2Q}\right);$$
$$b_{1s} = \frac{R_c}{2Q\tau};$$
$$b_{2s} = \left(\frac{R_c}{2Q\tau}\right) * \left(\frac{1}{\tau} + 2Q\Delta\omega\right);$$

τ = cavity damping time constant (s);
$\Delta\omega = \omega_0 - \omega_d$ = detuning frequency (rad/s);
ω_0 = cavity resonant frequency (rad/s);
ω_d = drive frequency (rad/s);
R_c = cavity shunt resistance (ohms);
Q = intrinsic cavity quality factor.

In general, these functions are of second order, with a complex pair of poles and a real zero. However, with no detuning ($\Delta\omega$=0), pole-zero cancellation occurs, and the functions revert to first order, with a single real pole.

The complete response of the cavity is then:

$$\begin{bmatrix} V_{ci}(s) \\ V_{cq}(s) \end{bmatrix} = [Z(s)] \begin{bmatrix} I_{ci}(s) \\ I_{cq}(s) \end{bmatrix};$$

where:

$$[Z(s)] = \begin{bmatrix} Z_c(s) & -Z_s(s) \\ Z_s(s) & Z_c(s) \end{bmatrix};$$

$V_{ci,q}(s)$ = in/quadrature phase cavity voltage;
$I_{ci,q}(s)$ = in/quadrature phase cavity current.

LOADED CAVITY RESPONSE

As described in Ref. [1], the terminal conditions that govern the interface between the drive line, intrinsic cavity and beam current are given by:

$V_c = V_f + V_r$;
$I_c = (V_f - V_r)Y_0 + nI_b$;
$V_{f,r}$ = forward/reverse drive line voltage;
V_c = cavity voltage;

Y_0 = drive line characteristic admittance;
n = beam coupling transformer ratio;
I_c= cavity current;
I_b = beam current.

These relations are represented in block diagram form in Fig. 1. Note that all signals are complex-valued.

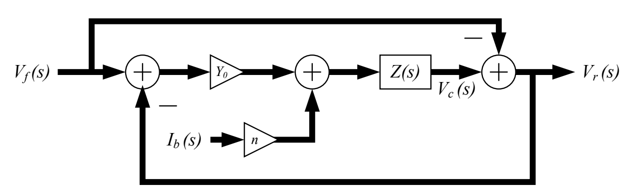

Figure 1: Operational block diagram of line/beam/cavity interactions.

While this operational model works well for simulation, algebraic equations may be more appropriate for analytical purposes. Thus, it can be shown that the following equations are equivalent to the loaded cavity model:

$$V_c = [I + Y_0 Z(s)]^{-1}[Z(s)][2Y_0 \quad n]\begin{bmatrix} V_f \\ I_b \end{bmatrix};$$
$$V_r = V_c - V_f.$$

* This work was supported in part by NSF award #1935994.
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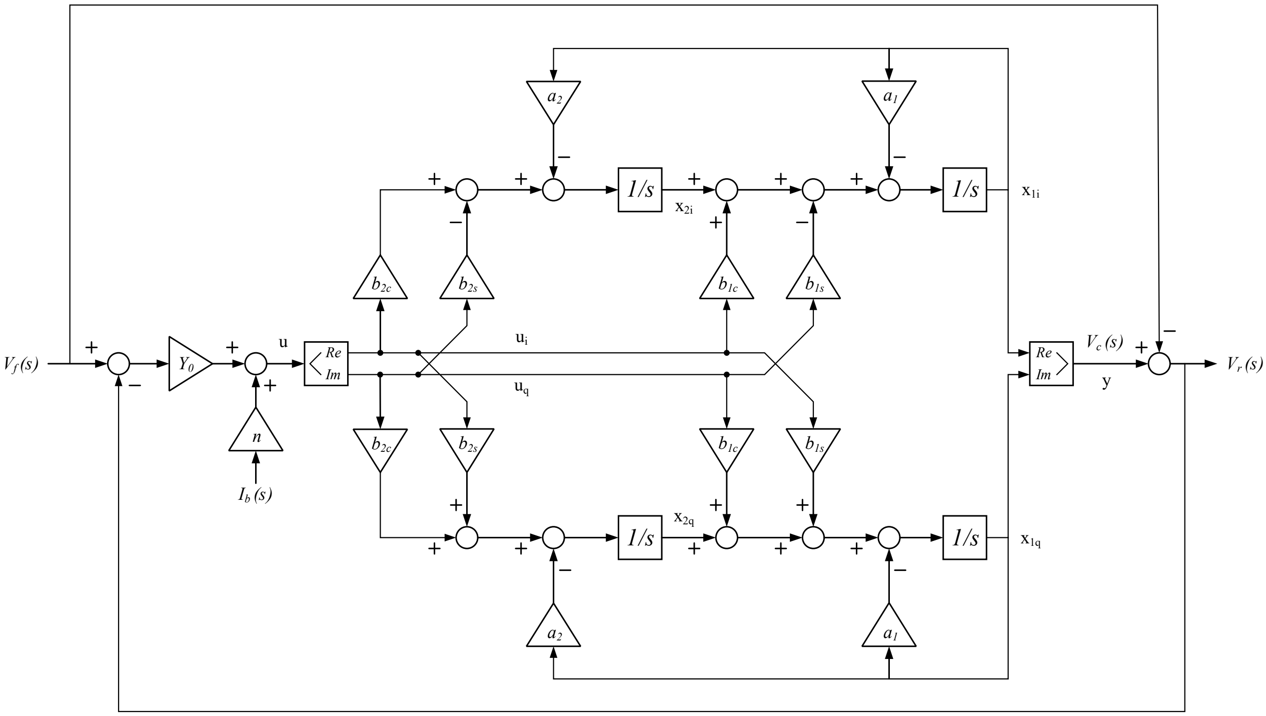

Figure 2: State-space realization block diagram of loaded cavity in second companion form.

STATE-SPACE MODEL

For control system design, a state-space model of the plant to be controlled is often required. Figure 2 shows one such realization in second companion form [2, 3]. The intrinsic cavity behavior is expressed in standard form:

$$\dot{x} = Ax + Bu,$$
$$y = Cx + Du,$$

where:

$$A = \begin{bmatrix} -a_1 & 1 & 0 & 0 \\ -a_2 & 0 & 0 & 0 \\ 0 & 0 & -a_1 & 1 \\ 0 & 0 & -a_2 & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} b_{1c} & -b_{1s} \\ b_{2c} & -b_{2s} \\ b_{1s} & b_{1c} \\ b_{2s} & b_{2c} \end{bmatrix};$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix};$$

$$D = 0;$$

$$x = \begin{bmatrix} x_{1i} \\ x_{2i} \\ x_{1q} \\ x_{2q} \end{bmatrix}.$$

Then with loading:

$$u = (V_f - V_r)Y_0 + nI_b;$$
$$V_c = y;$$
$$V_r = y - V_f.$$

The internal states (x_{2i}, x_{2q}) are not physically accessible, but this model can be a basis for design of a reduced-order asymptotic observer [4]. With the estimated internal states, full-state feedback can be implemented.

The design and implementation of a real-time analog simulator, utilizing an alternative canonical form, can be found in Ref. [5].

SIMULATION RESULTS

To illustrate the utility of the cavity model, several scenarios were simulated with the system of Fig. 2. and:

$$\tau = 20 \text{ us};$$
$$Q = 50{,}000;$$
$$\text{n} = 1.$$

Figure 3 shows the filling response of an overcoupled cavity ($\beta = R_c Y_0 = 2$) with no detuning ($\Delta\omega = 0$). RF power is applied at $t = 10$ µs. Note that the reflected power passes through zero momentarily, on the way to its final value.

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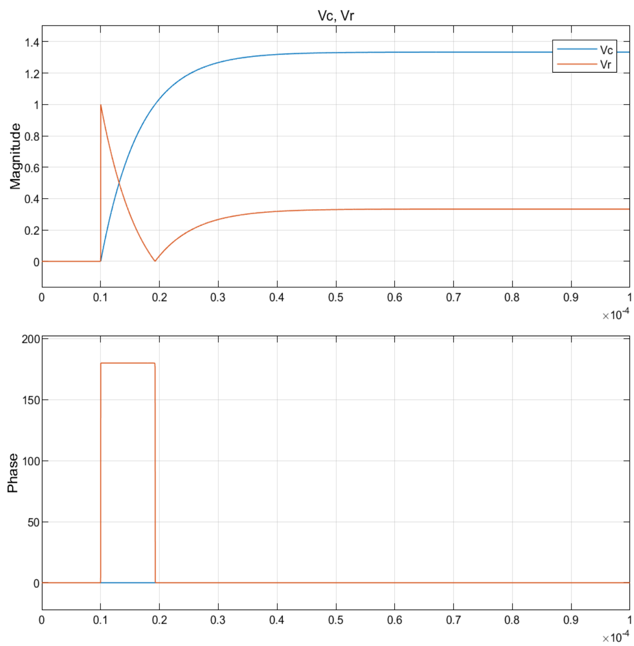

Figure 3: Response of overcoupled cavity.

When beam (20 mA @ -135 deg) is added at $t = 50$ us to a critically-coupled cavity ($\beta = 1$), the effect on cavity and reflected voltage is shown in Fig. 4.

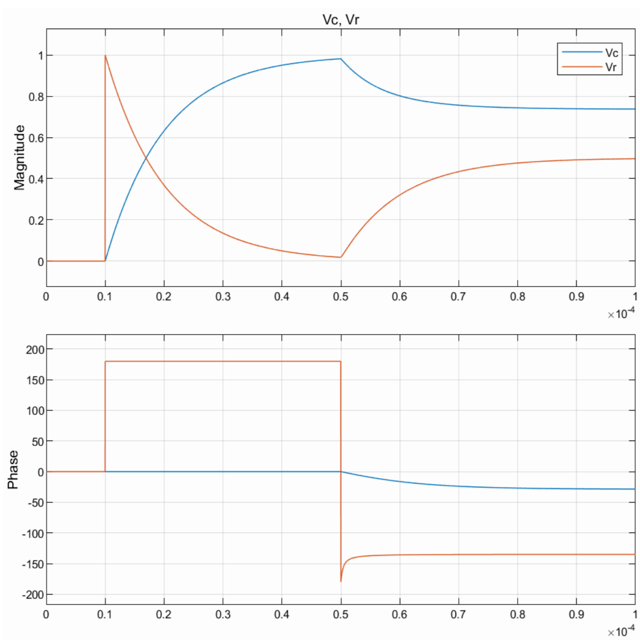

Figure 4: Response of critically-coupled cavity with beam loading.

A more complex response is obtained in Fig. 5. Here, an overcoupled cavity ($\beta = 4$) is driven detuned ($\Delta\omega = 1e6$ rad/s), and beam is applied (101 mA @ -99°). After final settling, the reflected power is zero, and the cavity appears to be matched. Figure 6 shows the cavity voltage response in the complex plane, where the transient oscillations of cavity settling are clear.

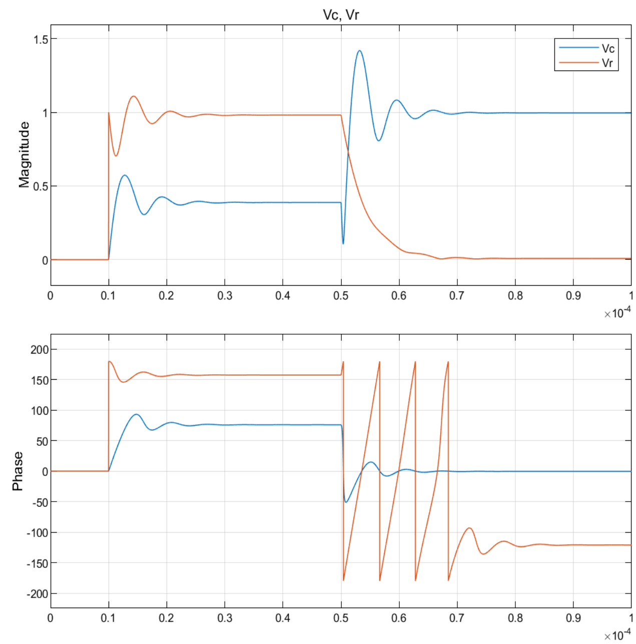

Figure 5: Overcoupled and detuned response.

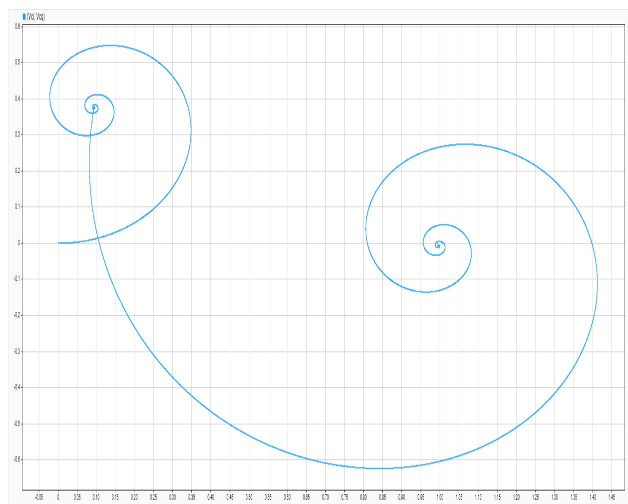

Figure 6: Complex cavity voltage settling.

Additional model responses, including multiple drive ports and parasitic modes, can be found in [1].

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