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THE IMPACT ON THE VERTICAL BEAM DYNAMICS DUE TO THE NOISE IN A HORIZONTAL CRAB CROSSING SCHEME*

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Abstract

Several recent and future colliders have adopted the crab crossing scheme to boost performance. The lower RF control noise of the crab cavities has been identified as one of the significant sources that impact the transverse beam quality in the crabbing plane. However, through beam-beam interaction and other coupling sources, the effect may also affect the non-crabbing plane. In this paper, we report the simulation observations of the beam dynamics in the non-crabbing plane in the presence of phase noise in the crab cavity.

INTRODUCTIONS

The RF crab cavity noise has been identified as the source of the transverse emittance growth of the crabbing plane in the hadron ring of a collider. A linear theory has been reported to calculate the emittance growth rates in the presence of the voltage and phase error of the crab cavity respectively [1]. A feedback system can be helpful to reduce the emittance growth [2]. Recently, the noise effect is tested in SPS crab cavity test and the discrepancy of noise-induced emittance growth between the experimental observation and the theory are found to be explained by the coherent tune shift due to the transverse coupling impedance [3].

The crab cavity noise-induced emittance growth is limited in the crabbing plane if no coupling effect presents. The linear coupling is an obvious source to affect the non-crabbing plane, while at least theoretically, the linear coupling can be corrected and the feedback system will be effective for the coupled emittance growth in the non-crabbing plane. There are also inevitable nonlinear coupling effects due to the multi-pole magnets between the crab cavity pairs or the beam-beam interaction. The nonlinear coupled emittance growth in the non-crabbing plane usually is small and can be ignored. However, the statement needs to be carefully checked in the future electron-ion collider (EIC). In EIC, the electron beam naturally has a very flat beam profile and the proton beam is designed to match at IP to achieve high luminosity. The transverse flatness r, defined as $r = \sigma_y/\sigma_x$ at the interaction point(IP), is designed to be about 0.09. Due to the strong radiation damping of the electron beam, only the hadron beam can be affected by the crab noises.

In this article, we will use a simplified beam-beam model to verify the vertical emittance growth in a horizontal crabbing scheme. The parameter in simulation is adopted from the parameter table of future EIC, as shown in Table 1. Both

Table 1: Crab Crosing Parameters of EIC

	Electron	Proton
Energy (GeV)	10	275
H.beam size (IP) (μm)	95	95
V.beam size (IP) (μm)	8.5	8.5
β_x (IP) (cm)	55	80
β_y (IP) (cm)	5.6	7.2
Bunch length (cm)	6	0.7
crossing angle (mrad)	25	

the crab noise in the hadron ring and electron ring will create z dependent transverse offset in the interaction region. We will only focus on the noise effect of the crab cavity in the hadron beam due to the page limit.

SIMPLIFIED TRACKING MODEL

In the simplified model, we use a linear one-turn matrix at IP to represent the entire hadron ring. The crab cavities in the hadron ring are located in the ideal location which has $\pi/2$ horizontal phase advance away from IP.

Since the electron beam is much shorter than the hadron beam, in this simplified beam-beam model, a short electron beam is assumed. To avoid the computationally expensive Faddeeva function in the Bassetti-Erskine formula, we use the flat beam approximation form as below:

$$\Delta p_y = \frac{N_e r_0 \sqrt{2\pi}}{\gamma \sigma_x}\left(e^{-\tilde{x}^2}\,\mathrm{erf}\,(\tilde{y}) - \frac{2r\tilde{y}}{\sqrt{\pi}} + \frac{4r\tilde{x}\tilde{y}}{\sqrt{\pi}}F\,(\tilde{x})\right) \quad (1)$$

$$\begin{aligned}\Delta p_x = &\frac{N_e r_0 \sqrt{2\pi}}{\gamma \sigma_x}\left(\frac{2F\,(\tilde{x})}{\sqrt{\pi}} - 2r\tilde{x}e^{-\tilde{x}^2}\left(\tilde{y}\,\mathrm{erf}\,(\tilde{y}) + \frac{e^{-\tilde{y}^2}}{\sqrt{\pi}}\right)\right.\\ &\left. + r^2\tilde{x}\left(1+2\tilde{y}^2\right) + r^2\left(2\tilde{y}^2 - \tilde{x}^2 - 4\tilde{x}^2\tilde{y}^2\right)F\,(\tilde{x})\right)\end{aligned} \quad (2)$$

where $\tilde{x}$ and $\tilde{y}$ are the transverse locations normalized by $\sqrt{2}\sigma_{x/y}$ respectively. $\sigma_{x/y}$ are the two transverse rms beam sizes at IP. In this limit, the two evaluations of the Faddeeva function are replaced by the calculation of the error function $\mathrm{erf}(x)$ and Dawson's integral $F(x)$. This simplification not only speeds up the model to allow a broader scan of the emittance growth with various noise levels and other parameters, but also explores clearer mathematical dependence in the limit of flat beam. For simplicity, the energy kick due to the symplectic condition is not included, which is not essential in understanding the effect of the crab cavity noise in a short time range.

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In this model, we will focus on the phase noise of the crab cavity since it is believed to be more damaging. The noise will be applied to the crab cavities on both sides. One crab cavity on each side is assumed. In reality, 4 crab cavities will be installed. If the noise from each cavity is uncorrelated, the requirement retrieved from this study can be relaxed by about a factor of 2. To better characterize the lower-level RF noises, the pink noise model is applied in the study.

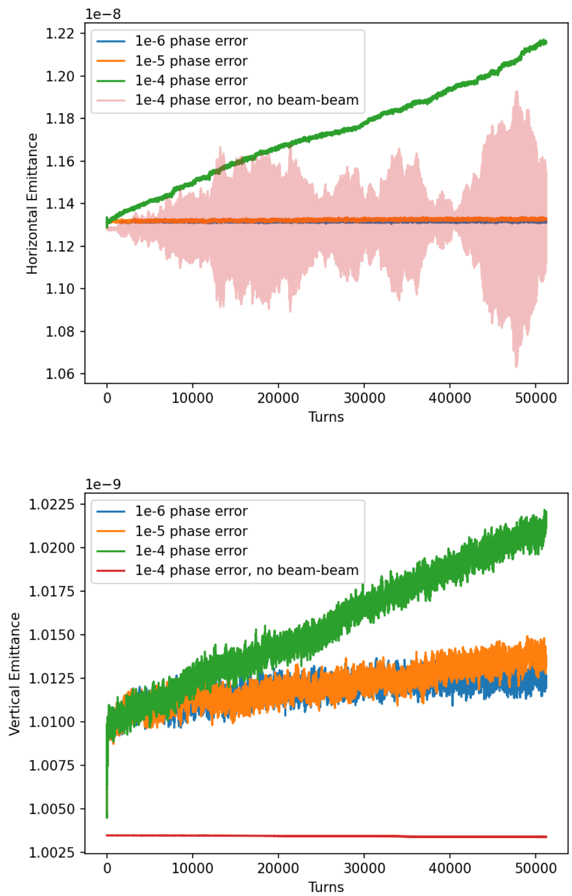

Figure 1: Comparison of horizontal (top figure) and vertical (bottom figure)emittance growth for various phase noise amplitude and with or without beam-beam interactions.

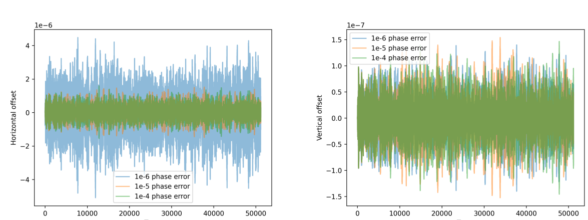

Figure 2: Comparison of horizontal (top figure) and vertical (bottom figure) offset for various phase noise amplitude.

In the simulation, 50K macro-particles are used in 50K turns tracking. Figure 1 illustrates the comparison of the horizontal and vertical emittance growth under 1×10^{-6}, 1×10^{-5} and 1×10^{-4} rad phase noise of crab cavity. In addition, with 1×10^{-4} rad phase noise, the results excluding beam-beam interaction are also included.

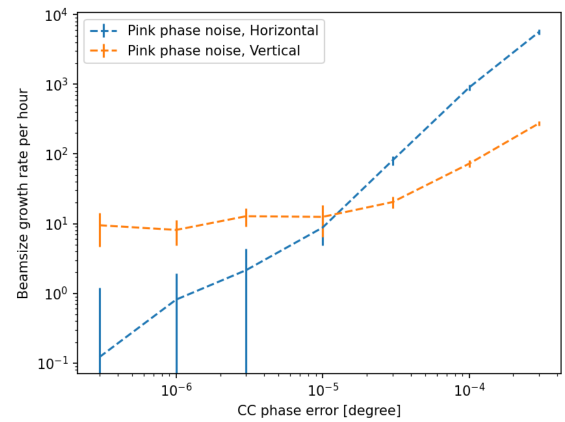

Figure 3: The horizontal and vertical emittance growth rate as function of the phase noise error of the crab cavities.

First, without the beam-beam interaction, only horizontal emittance change is observed. No dynamics are included to couple the motion to the vertical plane, therefore the vertical emittance remains constant during the simulation. The horizontal emittance, in this case, largely oscillates, due to the lack of decoherence in this simplified model.

With the beam-beam effect, both the horizontal and vertical emittance grow under the phase noise. For a state-of-art cavity phase noise level, 1×10^{-4} rad, the vertical emittance shows a significant growth rate, about 1/10 of the growth rate of the horizontal direction. However, the reasons for the emittance growth in the two transverse directions are different. As shown in Figure 2, the horizontal beam center oscillation increases as the noise level are higher, while the vertical centers stay at the same level. This is due to the fact that the horizontal noise is the longitudinal dependent dipole kicks while the vertical noise is the nonlinear kicks as:

$$\delta p_y \sim e^{-\tilde{x}^2} \tilde{x} \, \mathrm{erf}\,(\tilde{y}) \, \delta x(z) \tag{3}$$

where, δx can either caused by the phase noise of the hadron beam ($\delta x \sim \cos(k_{cc} z) \delta\phi$) or by the position offset noise of the electron at IP. Therefore, for a fix deviation δx, the strength of the kick is a odd function for both positions x and y and therefore the average kick vanishes.

It is worthwhile to note that, in the vertical direction, the numerical noise in beam-beam interaction also contributes to the emittance growth. This can be clearly demonstrated in Figure 3. The numerical error dominates the overall vertical emittance growth until the crab cavity's phase noise exceeds 1×10^{-5} rad level. Therefore in later studies, in order to minimize the effect of numerical study and reflect the current state-of-art level of controlling phase noise, we will fix the phase noise to be 1×10^{-4} rad.

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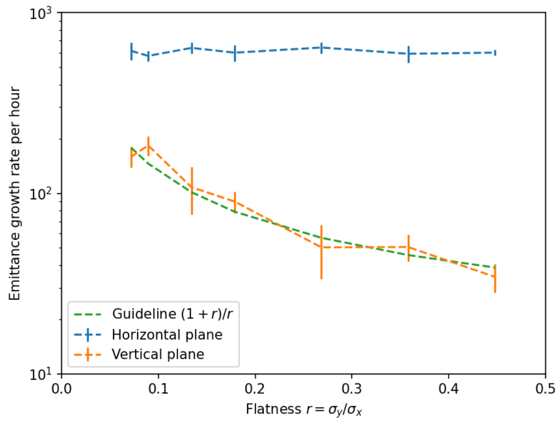

Figure 4: The horizontal and vertical emittance growth rate as function of the flatness of the beam transverse size at IP.

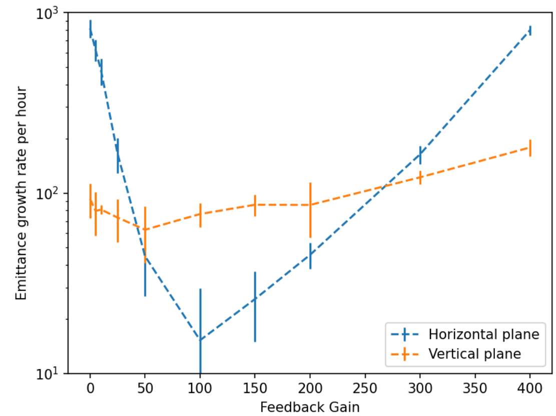

Figure 5: The horizontal and vertical emittance growth rate as function of the gain in the feedback system.

EFFECT OF THE FLATNESS

We are interested in whether the visible vertical emittance growth due to the crab cavity noise is due to the flat beam profile. Based on the parameters in Table 1, the vertical beam size is scaled to change the flatness r in both electron and hadron beams, while keeping their horizontal beam size at IP to be constant. Since the vertical effect arises due to the beam-beam effect, we choose to keep the beam-beam parameter constant while scaling the flatness. It is well-known that the beam-beam parameters $\xi_{x/y}$ can be calculated as:

$$\xi_{x/y} \sim \frac{\beta^*_{x/y}}{\sigma_{x/y}\left(\sigma_x + \sigma_y\right)} \tag{4}$$

where $\beta^*_(x/y)$ are beta waist at IP. To cancel the effect of changing flatness r, the two beta functions scale as

$$\beta^*_x \sim (1+r) \quad ; \quad \beta^*_y \sim r\,(1+r) \tag{5}$$

as a result, the rms emittance of two transverse plane should scales:

$$\epsilon_x \sim \frac{1}{1+r} \quad ; \quad \epsilon_y \sim \frac{r}{1+r} \tag{6}$$

Figure 4 indicates that the emittance growth rate in the horizontal plane, i.e. the crabbing plane, remains constant when the flatness changes from 0.07 to 0.45, while the vertical growth rate decreases. The vertical growth rate almost follows the reference line $\sim (1+r)/r$ which is inversely proportional to the design vertical emittance. This indicates that the time slope of the vertical emittance is almost a constant function of the flatness. The effect becomes more pronounced as the design vertical emittance shrinks.

POSSIBLE FEEDBACK SYSTEM

A feedback system can be adopted to suppress the emittance growth in the crabbing plane [2] which is the horizontal plane in the example above. The offset of each individual bunch can be detected by the pickup and the signal be feedback to the Low-level RF control of the crab cavity to adjust the phase. A simplified scheme is added to our model to test the effectiveness of suppressing emittance growth in the vertical plane. A very small rms pickup error of 1 micron is assumed to demonstrate the proof-of-principle in the simulation. The real-life pickup error for single bunch will be much larger.

Since the vertical offset will not increase due to the horizontal offsets, the feedback can only use the horizontal offsets as the input. Figure 5 shows that the horizontal growth rate can be effectively decreased. However, the vertical growth rate does not show significant improvement in the feedback process. Further studies are needed to find if an alternative feedback system is feasible to suppress the vertical emittance growth.

SUMMARY AND OUTLOOKS

We used a simplified model with a linear transfer map and weak-strong beam-beam force to explore the emittance growth of the non-crabbing plane. Despite the lack of real nonlinear lattice and numerical errors in the beam-beam kicks, the crab cavity noise-induced emittance growth in the vertical plane is observed. This effect is more pronounced in a flat beam profile. Since the vertical offset does not grow with the increasing noise level of the crab cavity, the feedback for the horizontal plane does show similar efficiency for suppressing the vertical growth in this preliminary study.

To get a more precise growth rate for a given noise level, a detailed simulation with realistic lattice is required [4]. However, element-by-element tracking is very time-consuming. Therefore, the simple model used in this article still has its value in parameter scanning and developing new countermeasures for vertical emittance growth.

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