Model parameters determination in EIC strong-strong simulation

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Electron-Ion Collider

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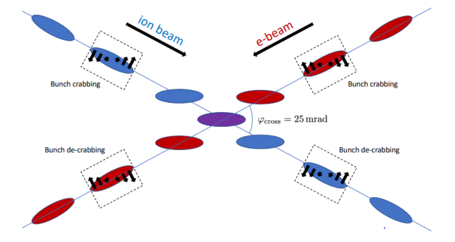
Introduction

- Large crossing angle 25 mrad
- Local crab crossing: upstream and downstream crab cavities to restore effective head-on collision to compensate geometric luminosity loss
- Large beam-beam parameters, $\mathrm{e} \sim 0.1, \mathrm{p} \sim 0.015$, combination never experimentally demonstrated
- Flat beam $\sigma_y/\sigma_x = 0.09$ to achieve highest e-p luminosity $10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

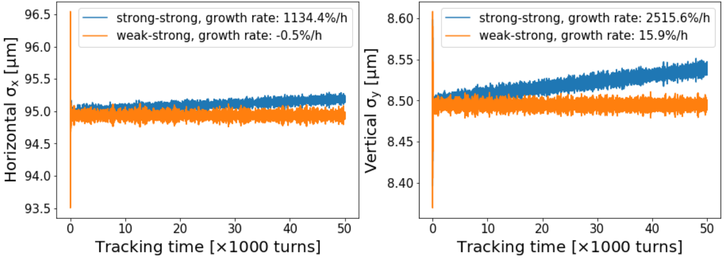


Parameter	unit	proton	electron
Circumference	m	3833.8451	
Particle energy	GeV	275	10
Bunch intensity	10^{11}	0.668	1.72
# of Bunches	-	1160	
Crossing angle	mrad	25	
β^* at IP	cm	80/7.2	45/5.6
Beam sizes at IP	µm	95/8.5	
Bunch length	cm	6	2
Energy spread	10^{-4}	6.6	5.5
Transverse tunes	-	0.228/0.210	0.08/0.06
Longitudinal tune	-	0.01	0.069
BB parameter	-	0.012/0.012	0.07/0.10
Luminosity	$\mathrm{cm}^{-2}\mathrm{s}^{-1}$	10^{34}	

Problem

Weak-strong vs strong-strong simulation

- The weak-strong model is used to study the single particle dynamics
- The strong-strong model is used to study the coherent motion
- The particle-in-cell (PIC) approach is widely used in strong-strong simulation. However, PIC is subject to numerical noise



It is important to understand the difference in case there is some coherent mechanism shadowed by the large numerical noise

Scaling law of macroparticles

Numerical errors in PIC based strong-strong simulation

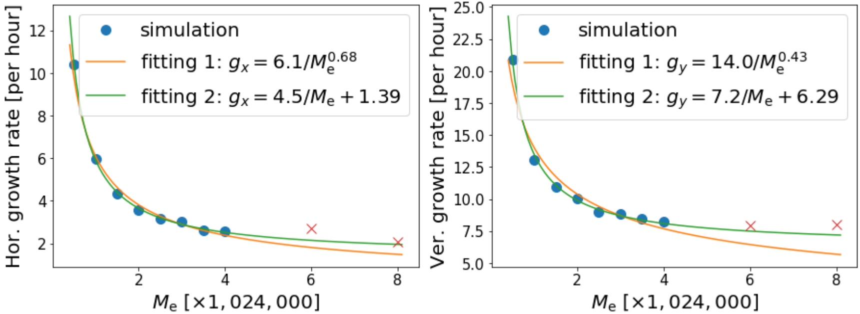
- Monte Carlo noise: number of macroparticles $\sim 10^6$, number of real particles $\sim 10^{11}$
- Transverse interpolation: (1) charge deposition on a finite grid, (2) field interpolation according to particle position in the grid
- Longitudinal interpolation: slice-slice collision instead of particle-slice collision to save computation time, $\phi(z) = w\phi(z_f) + (1-w)\phi(z_b)$

The numerical noise will cause the **particle diffusion** in phase space. More macroparticles can reduce the impact of numerical errors. If the beam size or emittance growth is purely determined by numerical noise, the theoretical scaling law is

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}t} \propto \frac{1}{M}$$

Scaling electron macroparticles M_e

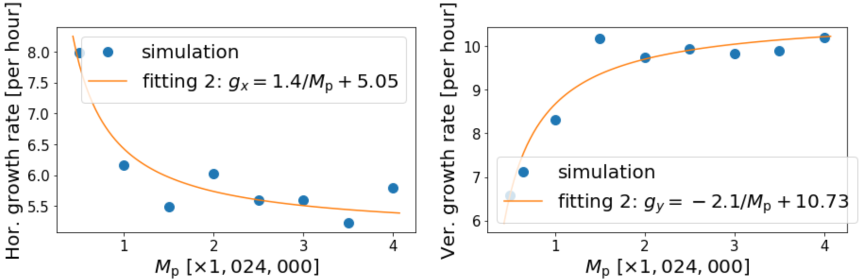
Proton growth rate with different M_e, circle: fitting, cross: validation



- The green curve is a better guess, $g_{x,y} = \frac{A_{x,y}}{M_\mathrm{e}} + B_{x,y}$
- The non-zero intercept $B_{x,y}$ needs further explanation

Scaling proton macroparticles M_p

Proton growth rate with different M_p

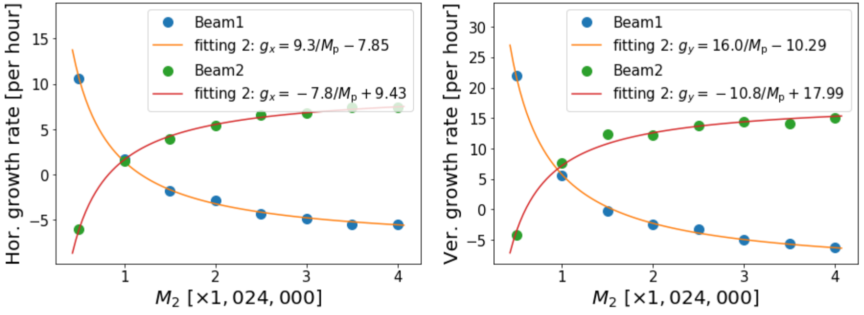


- More proton particles increase the vertical growth rate
- $1/M$ scaling law doesn't fit M_p

Hypothesis: there happens an emittance exchange which relates to the number of macroparticles. The synchrotron radiation prevents the exchange.

Scaling proton macroparticles M_p for $\mathrm{p}^- - \mathrm{p}$

- Replacing the electron beam with the anti-proton beam
- Turn off the synchrotron radiation and quantum excitation; the beam-beam parameter is reduced from 0.1 to 0.01. The working point is changed accordingly
- Other beam parameters and PIC setup are same as the electron beam



$1/M$ scaling law works good and hypothesis checked

Longitudinal slices

Pros of more longitudinal slices

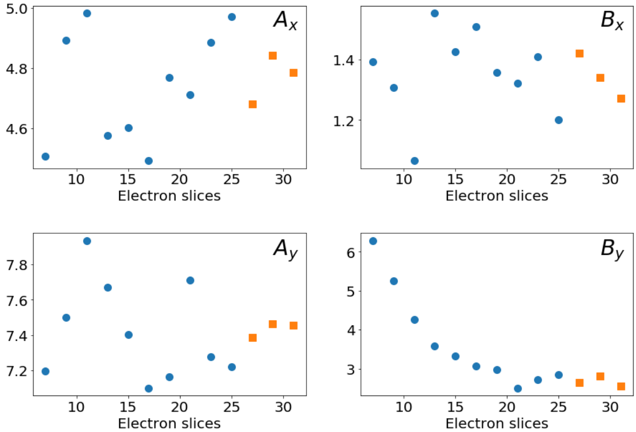
- More slices make the beam-beam force more smooth along the longitudinal direction
- More slices reduce the global numerical error: the integration is calculated by drift-kick model
- More slices reduce the longitudinal interpolation error

Cons of more longitudinal slices

- The number of macroparticles per slice decrease. The Monte Carlo noise increases
- The computation time roughly scales as the product of the number of slices in each beam

Longitudinal slices

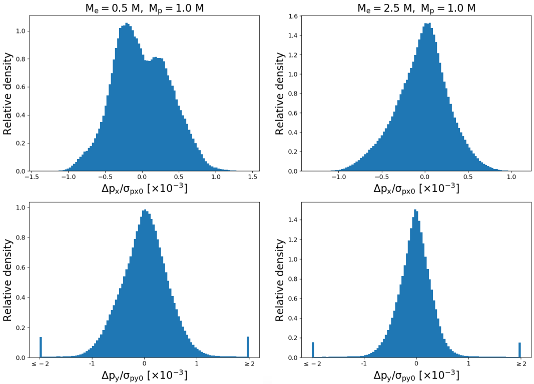
Fitting $A_{x,y}$ and $B_{x,y}$ for different number of slices $g_{x,y} = \frac{A_{x,y}}{M_e} + B_{x,y}$



B_y converges when the number of electron slices ≥ 21. There is no clear pattern for $A_{x,y}$ and B_x. With more slices, $B_x \sim 140\%/\mathrm{h}$, $B_y \sim 300\%/\mathrm{h}$

Histogram of PIC noise

- The energy of Beam1 (electron beam) is set to a large number 10^{100} eV so that Beam1 remains rigid during beam-beam interaction
- Comparing PIC field with analytic Basseti-Erskine formula



Histogram of PIC noise
Left: $M_\mathrm{e} = 0.5$ M
Right: $M_\mathrm{e} = 2.5$ M
Top: H error $\Delta p_x/\sigma_{px0}$
Top: V error $\Delta p_y/\sigma_{py0}$
The histogram area is normalized to 1

The ratio of large relative error in vertical plane is not reduced with more electron macroparticles. It explains why B_y is twice larger than B_x

Summary

- In PIC based strong-strong simulation, the horizontal growth rate $g_x \sim 1000\%/\mathrm{h}$, and the vertical growth rate $g_y \sim 2000\%/\mathrm{h}$
- When scaling electron macroparticles, both horizontal and vertical growth are well fitted to the scaling law $g_{x,y} = A_{x,y}/M_\mathrm{e} + B_{x,y}$ with $B_x \sim 140\%/\mathrm{h}$ and $B_y \sim 600\%/\mathrm{h}$
- There happens an emittance exchange which relates to the number of macroparticles. The synchrotron radiation prevents the exchange. To minimize this effect, we can keep $M_\mathrm{e}/M_\mathrm{p} = N_\mathrm{e}/N_\mathrm{p}$ in simulation
- B_y converges to $300\%/\mathrm{h}$ when the number of electron slices is ≥ 21
- When more electron macroparticles are used, the histogram horizontal relative error is a "Gaussian" shape. However, the ratio of large vertical error doesn't decrease

The growth in strong-strong simulation is dominated by numerical noise. The vertical emittance grows faster because of the PIC nature for flat beam.

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Thank you for your attention.