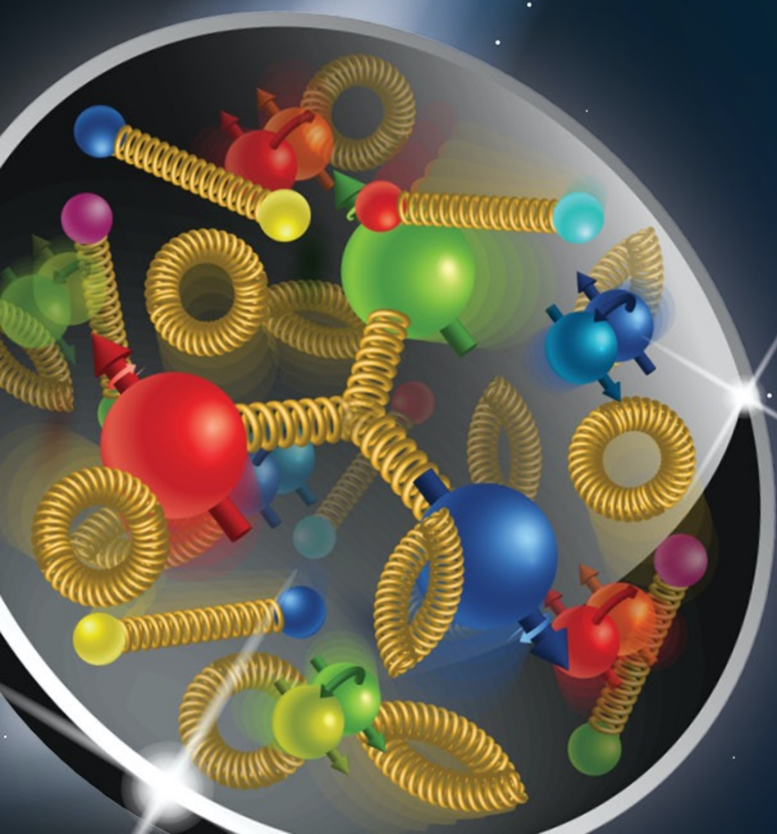
Tolerances of Crab Dispersion at the Interaction Point in the Hadron Storage Ring of the Electron-Ion Collider



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Electron-Ion Collider

BROOKHAVEN NATIONAL LABORATORY   Jefferson Lab   U.S. DEPARTMENT OF ENERGY | Office of Science

Outline

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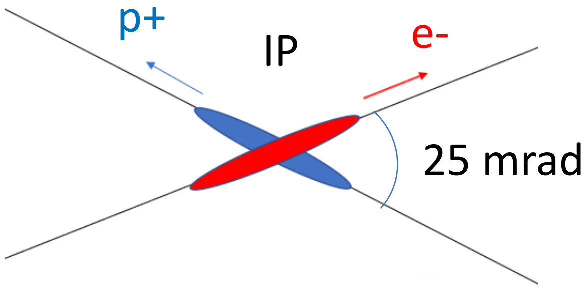
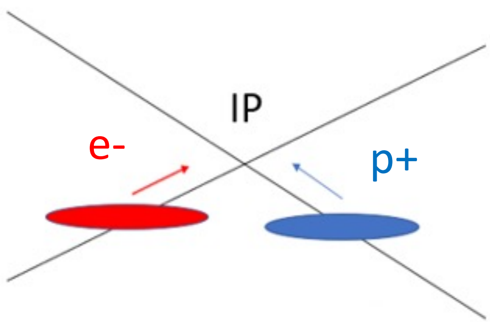
Crossing Collision in EIC

- Electron-Ion Collider (EIC) will collide polarized electron and hadron beams at center mass energies 20-140 GeV with maximum luminosities up to $1\times10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$.
- To achieve such a high luminosity, we adopt 1) high bunch intensities, 2) small transvers beam sizes at IP, 3) a large full crossing angle 25 mrad, and 4) a novel strong hadron cooling in the HSR.
- **To compensate the luminosity loss due to crossing angle, crab cavities are installed on both sides of IP** to tilt the bunch in the x-z plane by 12.5 mrad at IP to restore head-on collision condition.

$$L \propto \frac{1}{\sqrt{1+(\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}}$$

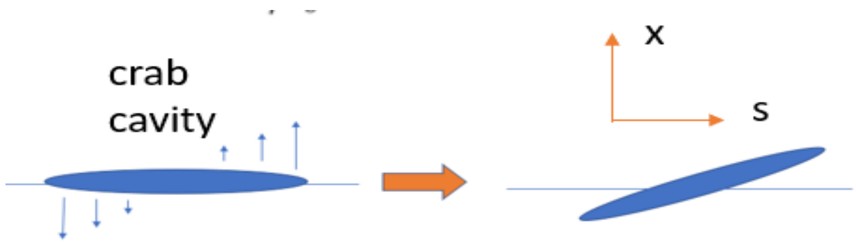


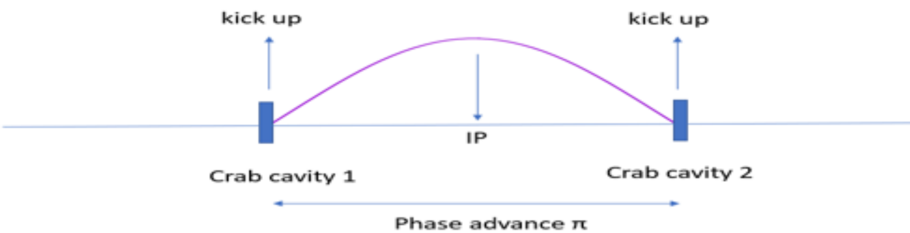
Without crabbing



With crabbing

Crab Cavity and Crab Dispersion





Crab cavity gives a z-dependent transverse kick, which generates a z-dependent transverse offset in downstream, ***Δx* = *d*$_c$ *z***.

EIC adopts **a local crabbing scheme** with two-bump of crab cavities. Ideally, the horizontal betatron phase advances to be 90 degrees between crab cavities and IP.

$$H_{crab} = \frac{qV}{E_s}\sin\left(\phi_s + \frac{\omega z}{c}\right)x,$$

$$\Delta p_x = -\frac{\partial H_{crab}}{\partial x} = -\frac{qV}{E_s}\sin\left(\phi_s + \frac{\omega z}{c}\right),$$

$$\Delta p_z = -\frac{\partial H_{crab}}{\partial z} = -\frac{qV}{E_s}\cos\left(\phi_s + \frac{\omega z}{c}\right)\frac{\omega}{c}x$$

linearizing

$$\Delta p_x = -K_c z$$

$$\Delta \delta = -K_c x$$

$$k_c = -\frac{qVw}{E_s c}$$

Closed orbit $x'' + K(s)x = \frac{\Delta BL}{(B\rho)}$

Momentum dispersion $x'' + K(s)x = \frac{\delta}{\rho}$

Crab Dispersion $x'' + K(s)x = k_c z$

$$d_{c,s} = \frac{\sqrt{\beta_{x,s}}}{2\sin\pi\mu}\oint k_c\sqrt{\beta_{x,s_0}}\cos(\pi\mu - |\Phi_x(s) - \Phi(s_0)|)ds$$

Crab Dispersion + Betatron Coupling

- If **there is X-Y coupling** around the ring, we need to solve a periodic 5-d solution for crab dispersion (**dx/dz, dx'/dz, dy/dz, dy'/dz**).
- **Sources of betatron coupling in colliders**: 1) detector solenoid, 2) residual roll angles of magnets, 3) residual vertical orbits in sextupoles, 4) residual roll angle of crab cavities, and 5) tilted ESR plane w,r.t. HSR plane.

$$\begin{pmatrix} D_x \\ Dx' \\ Dy \\ Dy' \\ 1 \end{pmatrix} = \begin{pmatrix} m11 & m12 & m13 & m14 & m15 \\ m21 & m22 & m23 & m24 & m25 \\ m34 & m32 & m33 & m34 & m35 \\ m44 & m32 & m43 & m44 & m45 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Dx \\ Dx' \\ Dy \\ Dy' \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} D_x \\ Dx' \\ Dy \\ Dy' \end{pmatrix} = (\mathbf{I} - \mathbf{M})^{-1} \begin{pmatrix} m15 \\ m25 \\ m35 \\ m45 \end{pmatrix}$$

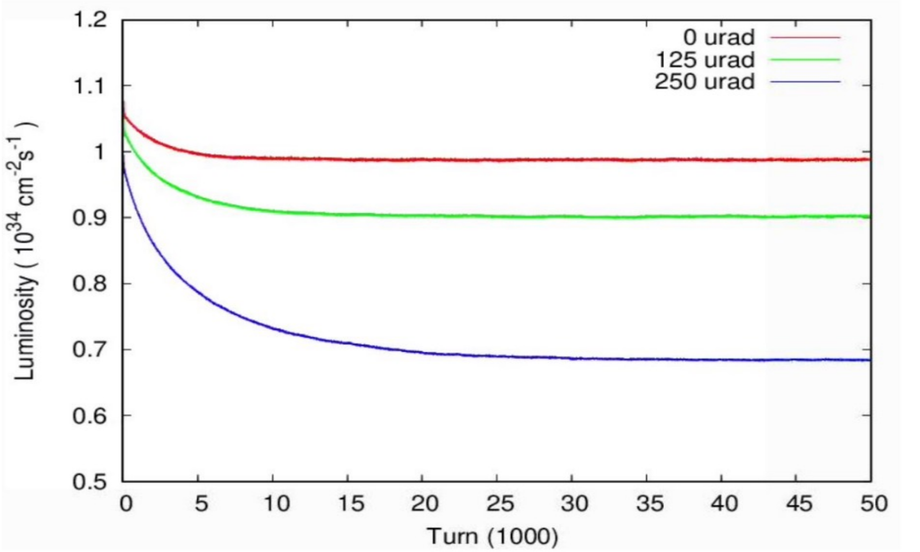
- **Vertical crab dispersion generates a z-dependent y offset**, that is, Δy = *(dy/dz) z,* along the bunch.
- **For ideal beam-beam collision at IP,** we would like to have crab dispersion at IP to be (*dx/dz, dx'/dz, dy/dz, dy'/dz*) =(12.5e-3, 0, 0,0). Non-zero *dy/dz* at IP generates an offset beam-beam interaction in vertical direction which may blow up vertical emittance and lead to a bad lifetime for proton beam.
- **In this following we estimate the tolerance of vertical crab dispersion (dy/dz) at IP with beam-beam interaction**.

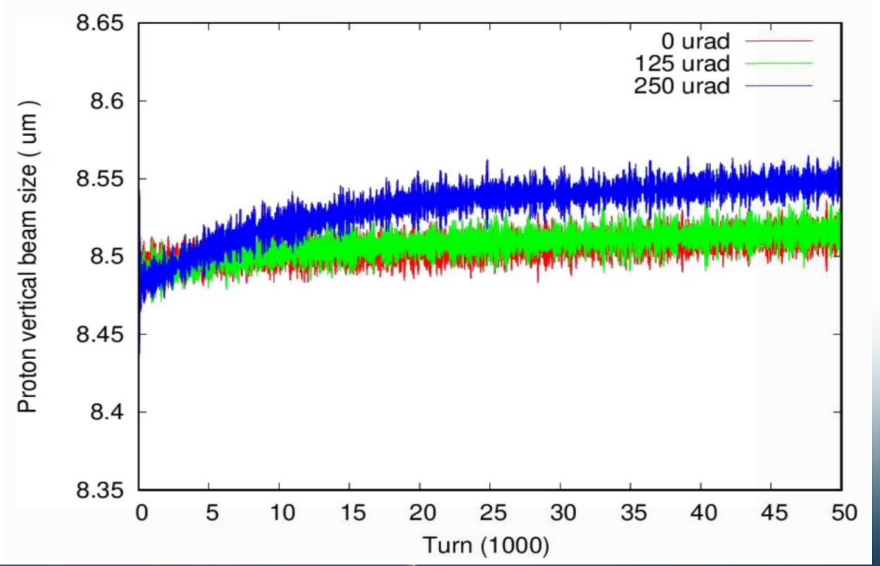
Strong-strong Simulation

- In the strong-strong model, **both beams are represented millions of macro-particles**. Beam-beam force is calculated with **PIC Poisson solver**. The ring is simply represented by a linear uncoupled 6×6 matrix.

- **Vertical crab dispersion at IP is created by** a virtual pair of 2-bump vertical crab cavities across the interaction region.

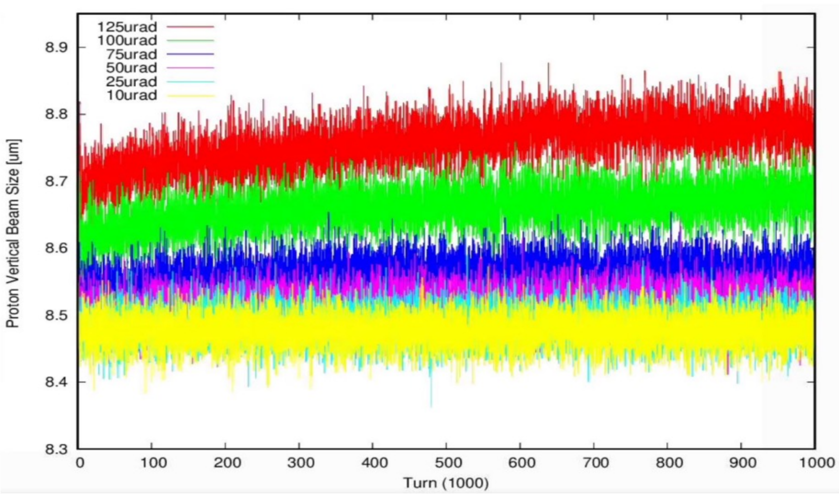
- Based on strong-strong simulation, **vertical crab dispersion *dy/dz* at IP should be less than 125 µrad, about 1% of design horizontal crab dispersion at IP.**



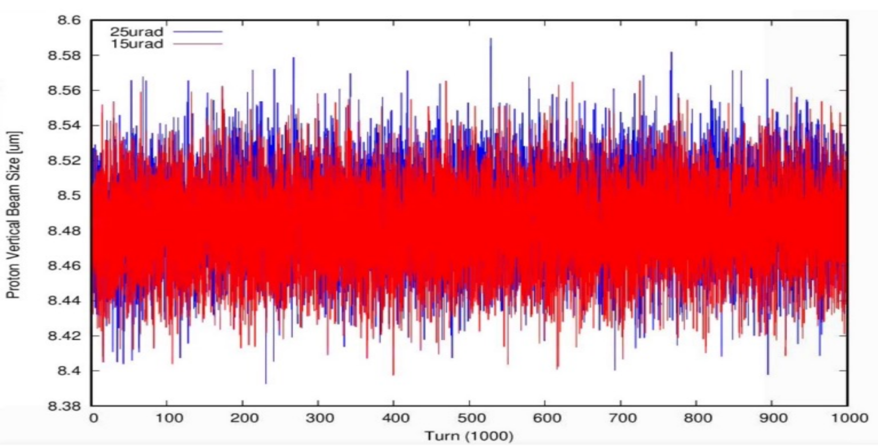


Weak-strong simulation

- In the weak-strong beam-beam simulation model, the proton bunch is represented by 30k macro-particles but electron bunch is represented by a rigid 6-d Gaussian charge distribution. The beam-beam force is analytically calculated with the Bassetti-Erskine formula. Particles are tracked up to 1-2 million turns.



dy/dz at IP (μrad)	Horizontal beam size growth rate (%/h)	Vertical beam size growth rate (%/h)
0	-0.4	-0.8
5	0.6	0.2
10	0.9	-3.9
15	-1.0	17.1
20	-0.1	-2.4
25	-0.4	-2.8
30	-3.6	23.3
40	-1.5	13.4
50	3.0	31.0
75	0.5	12.7
100	0.6	79.3
125	-4.2	100.1



- **Based on the above weak-strong simulation results, we set the tolerance of vertical crab dispersion at IP to be 25 μrad, which is only 0.4% of the design horizontal crab dispersion at IP.**

Dynamic Aperture Calculation

- We search the minimum amplitude in unit of sigma that protons can survive 1M turns with a 6-d element-by-element symplectic tracking. Therefore, the **complete HSR lattice is used**. Storage RF and crab cavities are included.
- Vertical crab dispersion at IP is introduced in the same way as early strong-strong & weak-strong simulations. **For each *dy/dz* at IP, we track10 seeds of nonlinear magnetic field errors** in the interaction region.

dy/dz at IP (μrad)	DA-Min (σ)	DA-Max (σ)	DA-Ave (σ)	DA-RMS (σ)
0	6.2	10.2	8.0	1.2
5	6.2	9.8	7.5	1.0
10	5.6	10.8	8.2	1.4
20	5.2	11.6	7.8	1.9
30	5.6	10.4	8.0	1.4
40	4.8	11.4	7.8	1.8
50	5.8	10.4	7.9	1.1
60	4.8	8.6	6.7	1.3

- If we assume that acceptable minimum dynamic aperture should be higher than 5 σ to guarantee a sufficient proton lifetime, **then the tolerance of vertical crab dispersion at IP will be about 20 μrad.**

Discussions

- We also studied **the tolerances for dx'/dz and dy'/dz at IP**. The tolerance for dx'/dz at IP is found about $1 \times 10^{-4} m^{-1}$, while the tolerance for dy'/dz at IP about $1 \times 10^{-5} m^{-1}$.
- We assumed even tighter crab dispersion tolerances at IP if there are **combined dy/dz, dx'/dz, and dy'/dz at IP**, which needs to be studied.

- **We also estimated the residual vertical crab dispersion (*dy/dz*) at IP** with the latest HSR design lattice:

1) About 1 μrad due to a random RMS 100 μrad roll errors to all quadrupoles in the HSR, 2) About 0.2 μrad due to a roll error of 200 μrad for all crab cavities on one side of IP.

- **For future EIC operation**, we need to have a robust algorithm and a reliable online system to compensate the residual vertical crab dispersion at IP. Skew quadrupole wires to be appended on IR quadrupoles. They are to be used for compensation of both systematic and residual vertical crab dispersion at IP.

Summary

- We studied the tolerances of vertical crab dispersion at the interaction point in the Hadron Storage Ring of the EIC. Strong-strong, weak-strong beam-beam simulations, together with dynamic aperture calculation are carried out for this purpose.
- Based on the simulation results with weak-strong mode and dynamic aperture calculation, the vertical crab dispersion tolerance at IP is about 20-25 μrad, which is only about 0.4% of design horizontal crab dispersion.
- For the future EIC operation, we need to have a reliable algorithm and a robust online system to compensate both systematic and residual vertical crab dispersion at IP.
- More simulation studies on tolerances of crab dispersion at IP and online compensation system design are in progress.

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