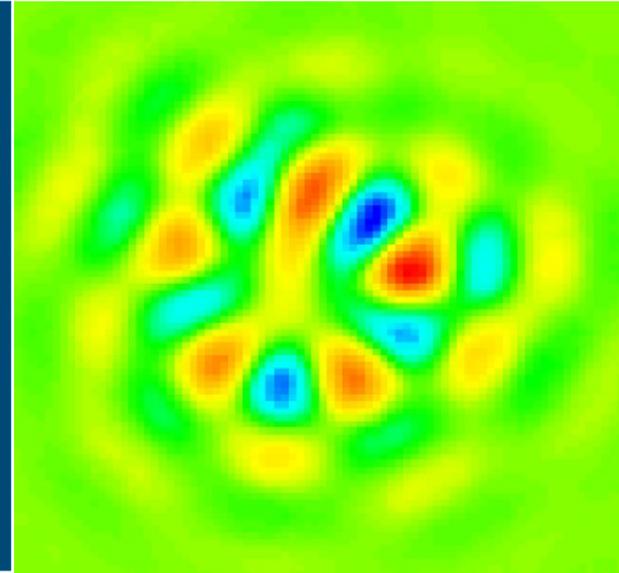


# Storage Ring Tracking Using Generalized Gradient Representations of Full Magnetic Field Maps



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North American Particle Accelerator Conference  
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# Outline

- Brief introduction to the generalized gradient representation of the magnetic field
- Overview of tools for using generalized gradients in eLegant
- Applications to APS upgrade modeling
  - Lambertson septum leakage fields
  - Building a generalized gradient lattice of the APS-U
  - Comparisons of tracking predictions with conventional methods
  - Using generalized gradients to validate improved hard edge and fringe models
- Conclusions and future directions

These results were obtained by Michael Borland and myself, with coding assistance from Bob Soliday.

Computations used the LCRC Bebop cluster at ANL and the weed cluster at ASD.

# Generalized gradient representation of the magnetic field

- A. Dragt and colleagues have developed the generalized gradient representation of magnetic field<sup>[1,2,3]</sup>
- The generalized gradient expansion represents  $\mathbf{B}$  using a power series in the transverse coordinates
  - Coefficients of the power series are  $z$ -dependent “multipoles” and their derivatives

$$A_z = \underbrace{-xC_1(z)}_{\text{Dipole}} - \underbrace{(x^2 - y^2)C_2(z)}_{\text{Quadrupole}} - \underbrace{(x^3 - 3xy^2)C_3(z)}_{\text{Sextupole}} + \frac{3}{8}(x^3 + xy^2)C_1''(z) + \dots + \text{Skew terms}$$

$$B_r = \sum_{m=1}^{\infty} \sum_{\ell=0}^{\infty} \frac{(-1)^\ell m!(2\ell + m)}{4^\ell \ell!(\ell + m)!} r^{2\ell+m-1} \left\{ C_{m,s}^{[2\ell]}(z) \sin(m\phi) + C_{m,c}^{[2\ell]}(z) \cos(m\phi) \right\} + \sum_{\ell=1}^{\infty} \frac{(-1)^\ell 2\ell}{4^\ell \ell!\ell!} r^{2\ell-1} C_{0,c}^{[2\ell]}(z)$$

$$B_\phi = \sum_{m=1}^{\infty} \sum_{\ell=0}^{\infty} \frac{(-1)^\ell m! m}{4^\ell \ell!(\ell + m)!} r^{2\ell+m-1} \left\{ C_{m,s}^{[2\ell]}(z) \cos(m\phi) - C_{m,c}^{[2\ell]}(z) \sin(m\phi) \right\} \quad B_z = \sum_{m=0}^{\infty} \sum_{\ell=0}^{\infty} \frac{(-1)^\ell m!}{4^\ell \ell!(\ell + m)!} r^{2\ell+m} \left\{ C_{m,s}^{[2\ell+1]}(z) \sin(m\phi) + C_{m,c}^{[2\ell+1]}(z) \cos(m\phi) \right\}$$

- The generalized gradient representation can be computed from measured or simulated magnetic field data
- Generalized gradient representations of magnetic fields enjoy a number of nice properties
  - Provides an analytic expression of  $\mathbf{B}$  with  $\nabla \cdot \mathbf{B} = 0 \rightarrow$  Symplectic tracking is possible
  - The vacuum curl equation  $\nabla \times \mathbf{B} = 0$  to a high order in the particle coordinates
  - Errors in the original field map are naturally smoothed

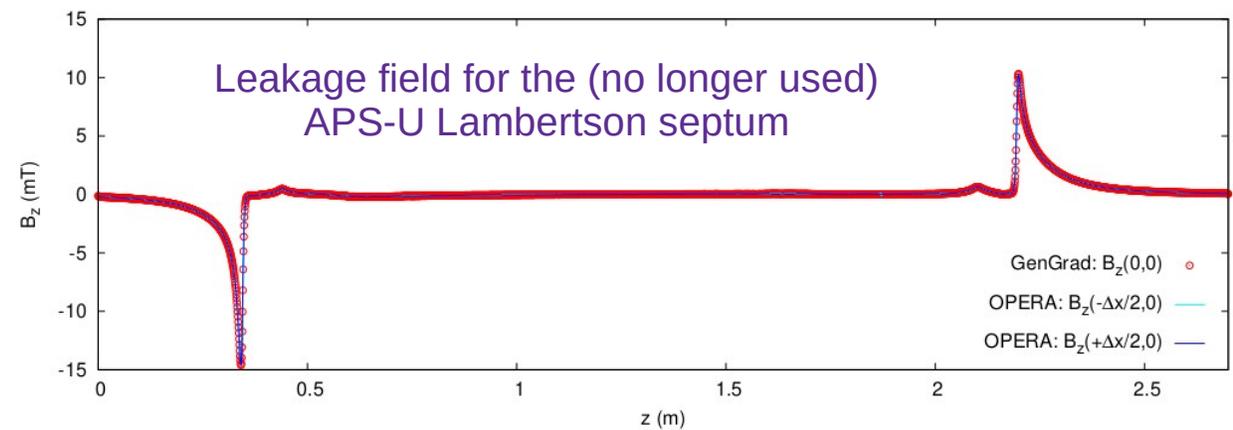
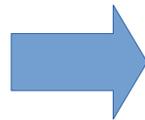
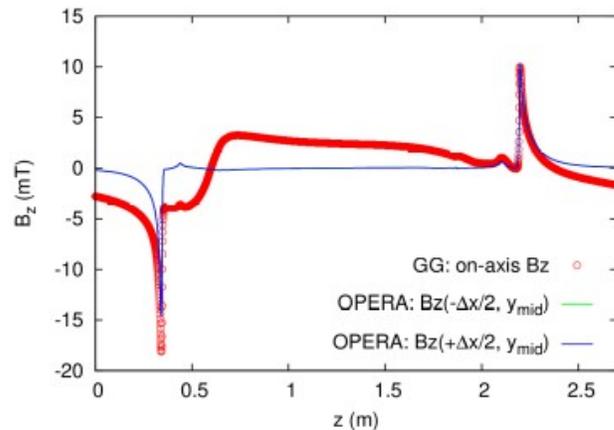
[1] M. Venturini and A. Dragt. “Accurate computation of transfer maps from magnetic field data,” Nucl. Instrum. Methods Res. A **427**, 387 (1999).

[2] A. J. Dragt. *Lie Methods for Nonlinear Dynamics with Applications to Accelerator Physics*. University of Maryland, College Park, 2020.

[3] C. E. Mitchell. “Calculation of Realistic charged-particle transfer maps.” PhD thesis, University of Maryland, College Park (2007).

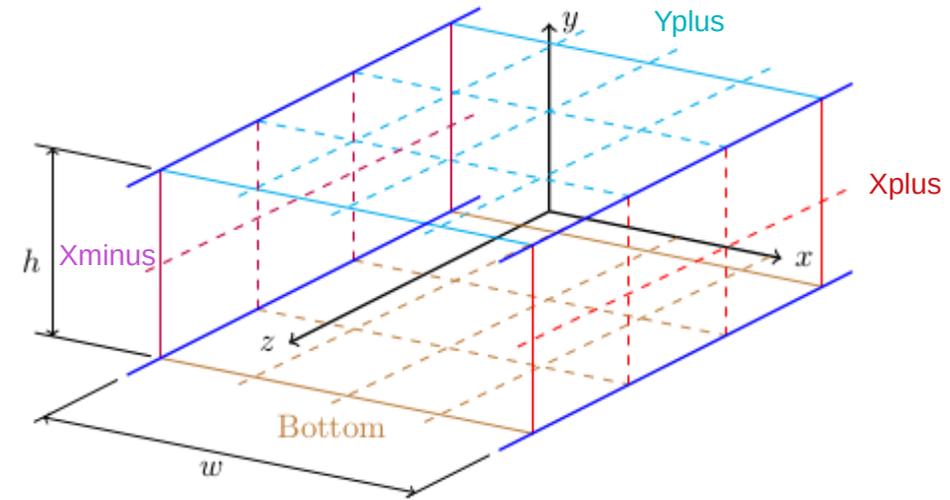
# Magnetic boundary data define the generalized gradients

- Published computational techniques use the normal component of  $\mathbf{B}$  on a generalized cylinder<sup>[1,2,3]</sup>
  - Orthogonal functions define bases in circular, elliptical, and rectangular cylinder
  - Solutions converge rapidly and also smooth any noise/errors in the boundary data
    - GG-"true solution" is a harmonic function whose maximum must lie on the boundary
- If the field has  $B_z \neq 0$  on-axis (solenoidal component), we found that  $B_z$  on the boundary is also needed



# Companion programs<sup>[5]</sup> use field data to compute the generalized gradient expansion for elegant tracking

- computeCBGGE uses the field data on the surface of a circular cylinder using equations from [1]
- computeRBGGE uses the field data on the surface of a rectangular cylinder using equations from [3]
- Both programs have several common features
  1. Choice of computing the normal, skew, or both field components
  2. Automated routine that finds the number of multipoles and derivatives to best match data
  3. Parallel computing using OpenMP
  4. Output files in a format suitable for the BGGEXP tracking element in elegant [4]



[5] M. Borland, R. R. Lindberg, R. Soliday, and A. Xiao, "Tools for Use of Generalized Gradient Expansions in Accelerator Simulations," in Proc. IPAC'21, pp. 253

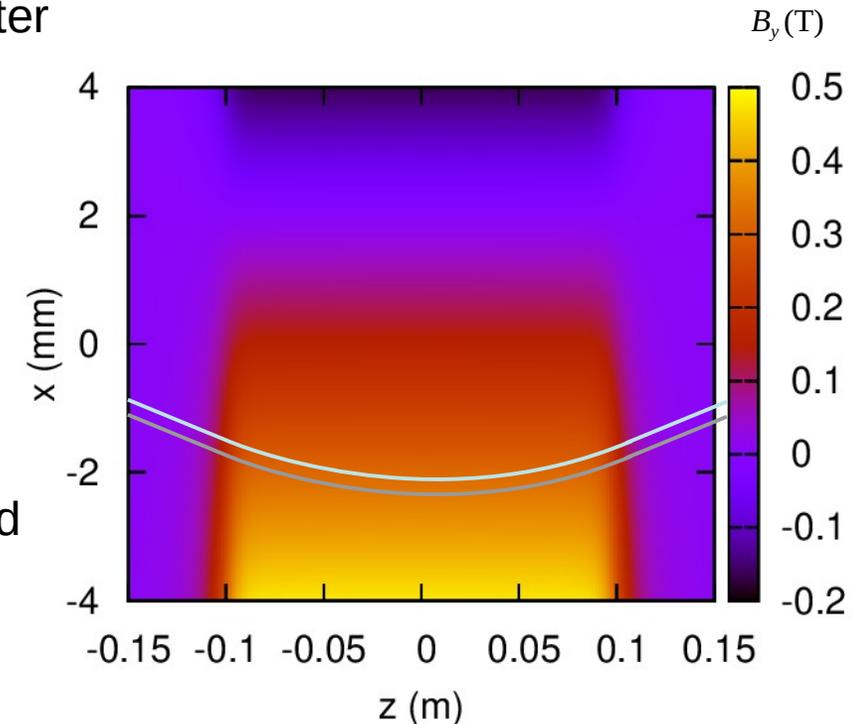
[1] M. Venturini and A. Dragt. "Accurate computation of transfer maps from magnetic field data," Nucl. Instrum. Methods Res. A **427**, 387 (1999).

[3] C. E. Mitchell. "Calculation of Realistic charged-particle transfer maps." PhD thesis, University of Maryland, College Park (2007).

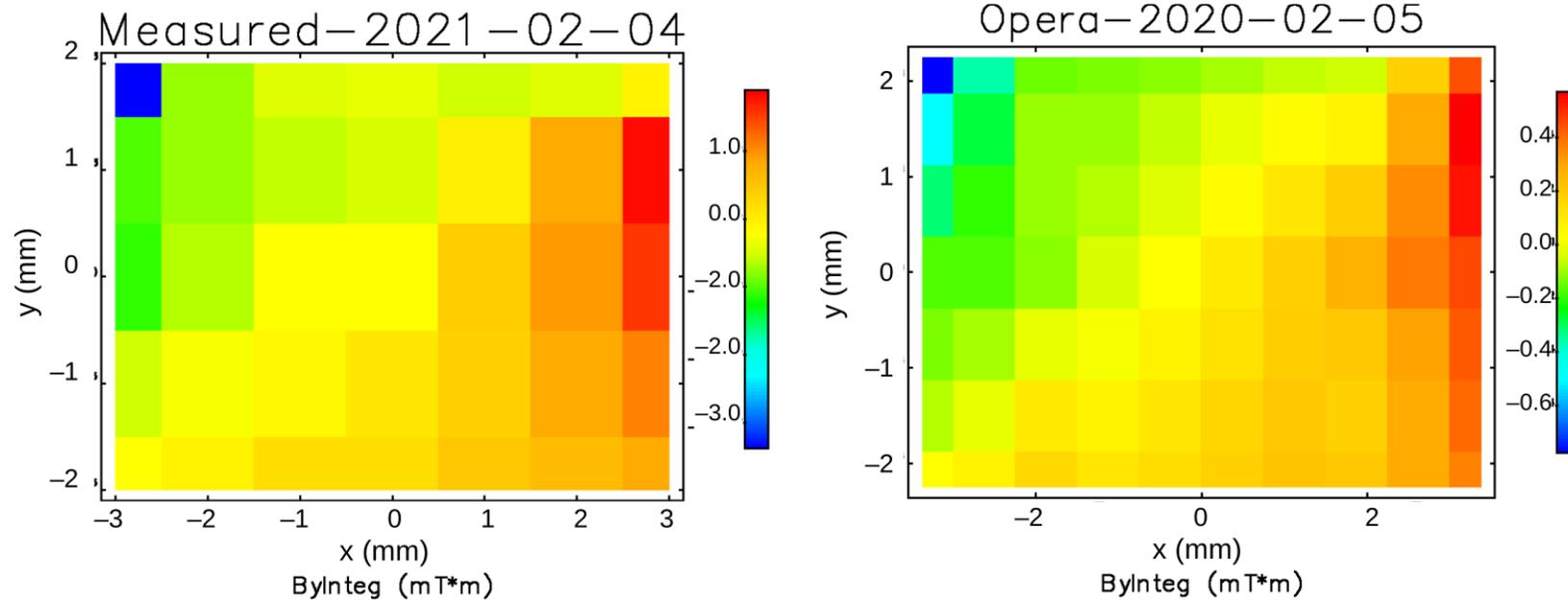
[4] M. Borland, "elegant: A Flexible SDDS-Compliant Code for Accelerator Simulation", LS-287 (2000).

# A brief overview of the BGGEXP element in elegant

- BGGEXP integrates particles through a field described by generalized gradients
  - Symplectic integrator using implicit midpoint method
    - Evaluates the vector potential  $A$  and updates the coordinates at locations between the data
    - Requires iteration for convergence
  - Nonsymplectic predictor-corrector
    - Explicit, only needs  $B$ -field components → over 3 times faster
- Tracking through quadrupoles, sextupoles, wigglers, etc. is relatively straightforward
- Tracking through dipoles is a bit trickier
  - Define input and output planes
  - Ensure correct bending angle using field-scaling parameters
- Tracking through gradient dipoles requires careful setup
  - Small changes in initial  $x$  will change the integrated bending field
  - Iterated adjustments in the strength and/or positional offsets is typically needed



# Application to modeling the septum leakage field



- Prototype APS-U Lambertson septum magnet<sup>[6]</sup> was built by FNAL and measured in May 2021
- Measurements by M. Kasa, *et al.* showed similar field map profiles to simulations, but with integrated leakage fields ~4 times larger than simulation predicted (error in construction)
- Previous studies indicated that a leakage field of this size could reduce injection efficiency and lifetime<sup>[7]</sup>
- We studied this issue using a generalized gradient field model derived from measurements

[6] M. Abliz *et al.* "A concept for canceling the leakage field inside the stored beam chamber of a septum magnet" NIM A **886**, 7 (2017).

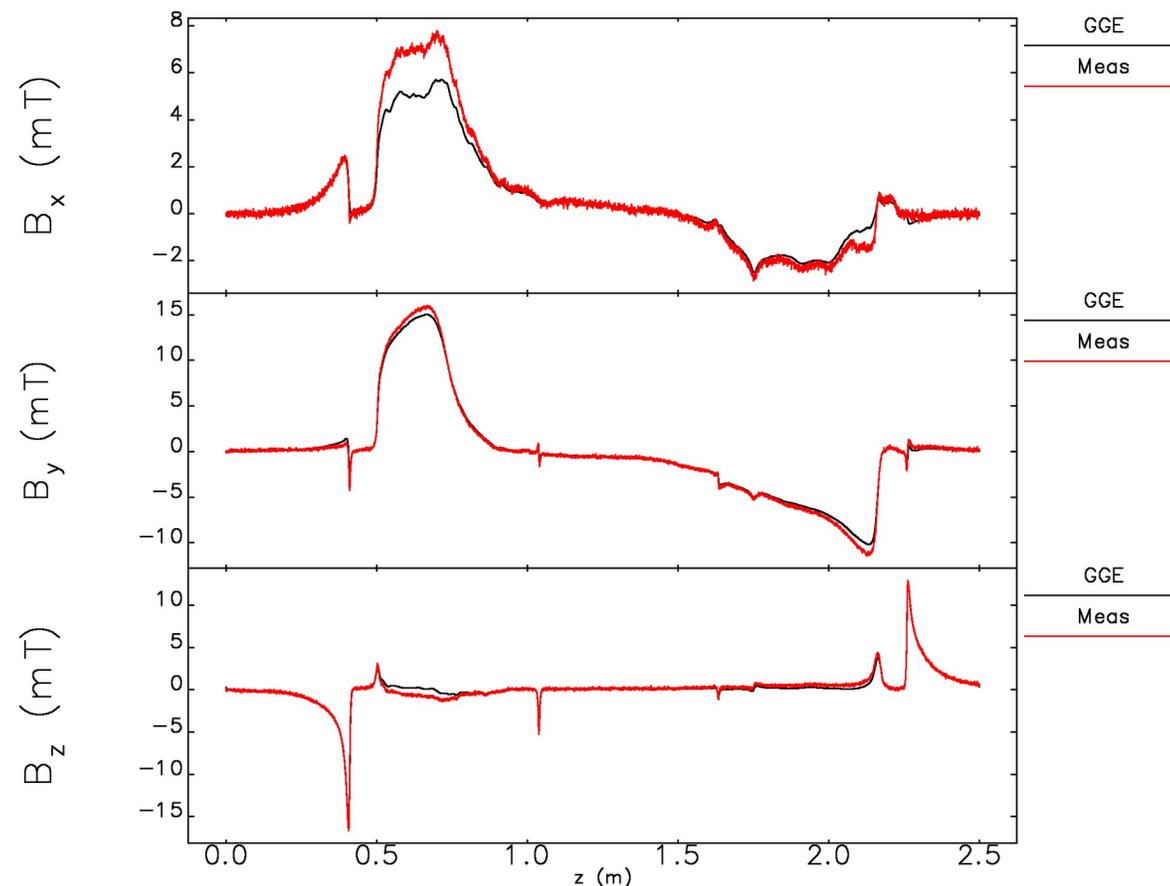
[7] A. Xiao *et al.* Private communication.

# Generalized gradient expansion (GGE) shows some deviations from the field measurements

- compute RBGGE found optimal “fit” using 6 multipoles and 2 derivatives
  - Rms “fit” error is 9X the 83  $\mu\text{T}$  measurement error
- On-axis  $B_y$  and  $B_z$  match well, but  $B_x$  does not
- Field differences appear in all components off axis

Component	RMS error	Largest error
$\Delta B_x$	0.55 mT	5.84 mT
$\Delta B_y$	0.41 mT	4.72 mT
$\Delta B_z$	0.20 mT	6.88 mT

- Inserting BGGEXP model into the APS-U lattice shows no significant harmful effects
  - Prior results based on kickmaps appear to have been misleading



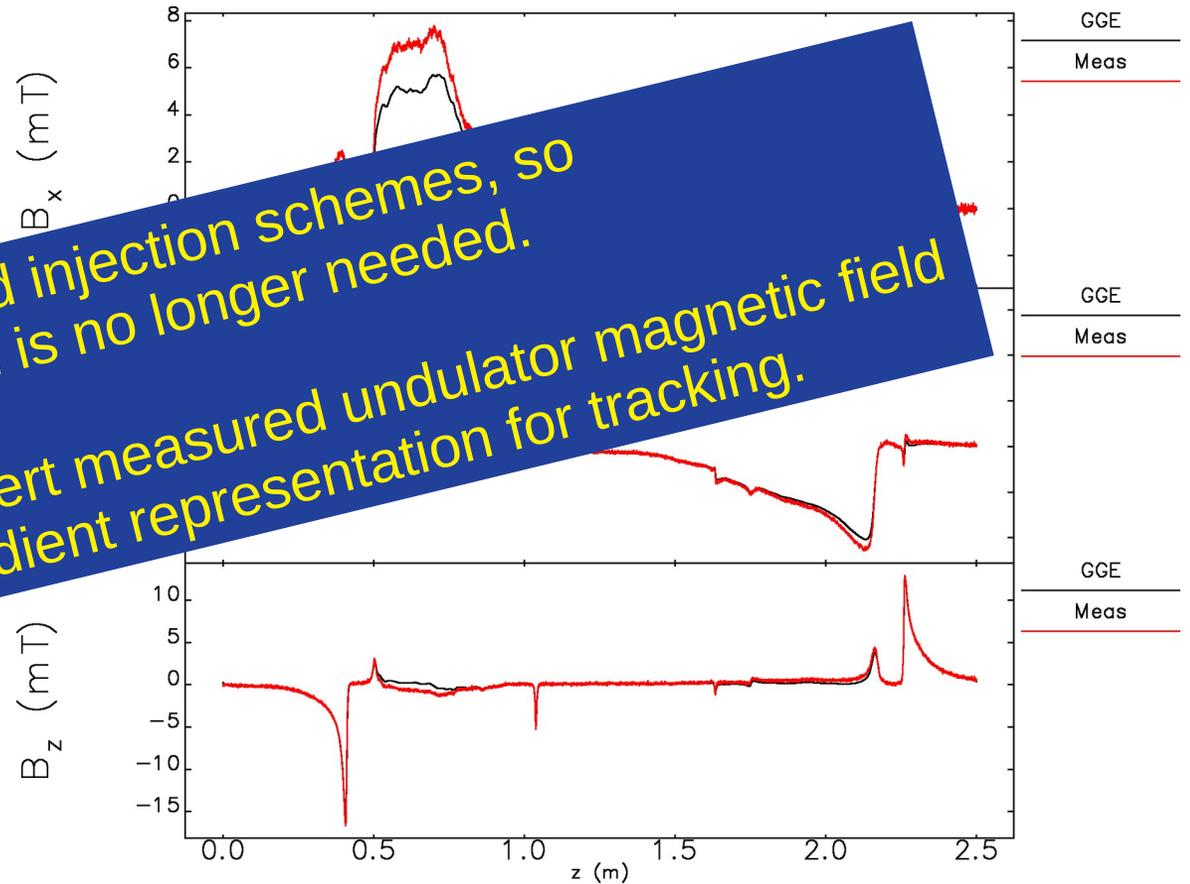
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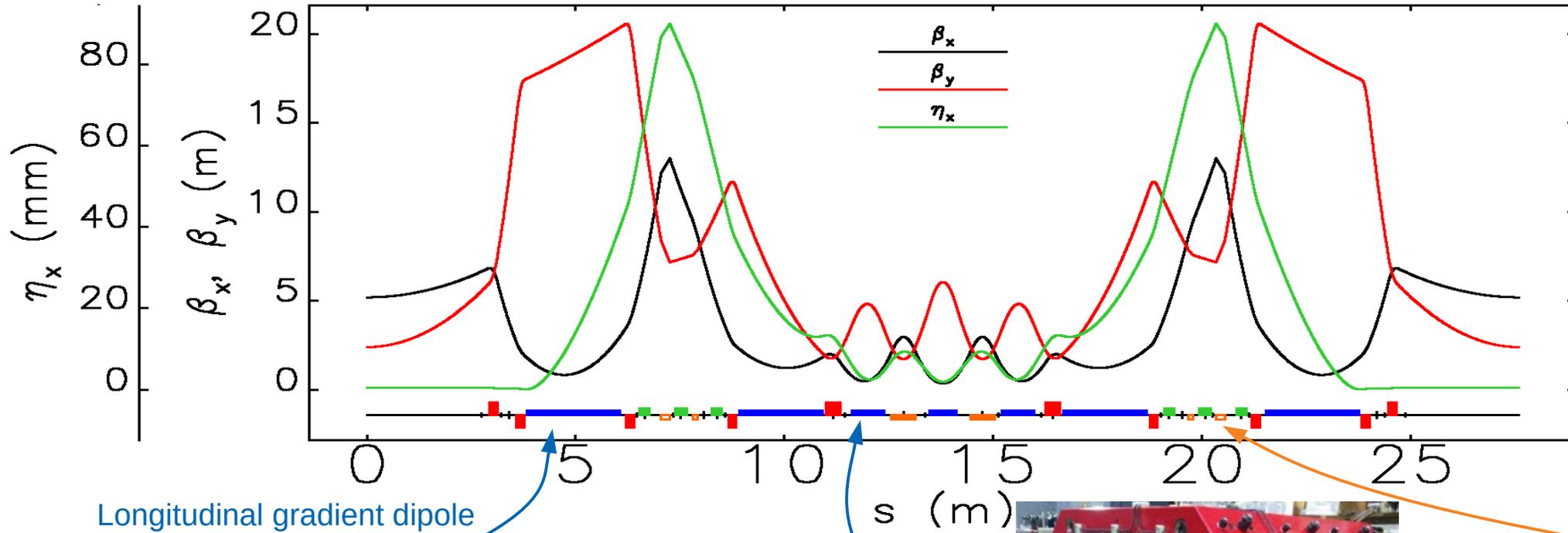
Component	$B_x$	$B_y$	$B_z$
$\Delta B_x$			
$\Delta B_y$			
$\Delta B_z$			

- Inserting  $B_x$  into the APS-U lattice shows no significant harmful effects
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We have since changed injection schemes, so Lambertson septum is no longer needed. Presently, we are working to convert measured undulator magnetic field data into a generalized gradient representation for tracking.



# APS-U uses a hybrid 7+BA lattice<sup>[9]</sup> to achieve $\epsilon_x = 42 \text{ pm}$ <sup>[10]</sup>



Dipoles  
Reverse bend dipoles  
Quadrupoles  
Sextupoles

Longitudinal gradient dipole

Transverse gradient dipole

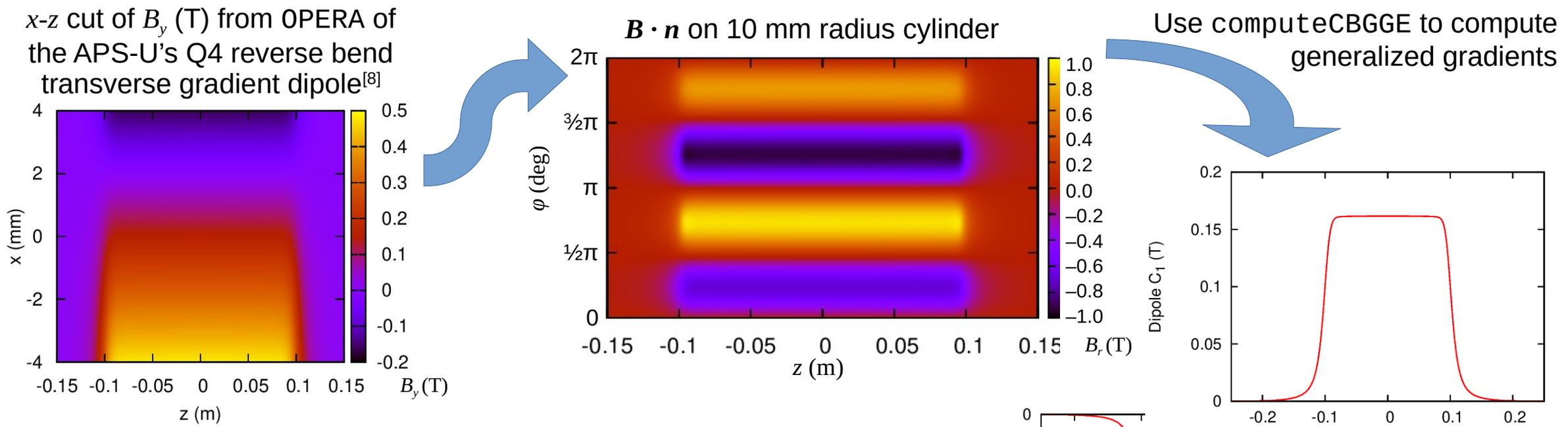
Transverse gradient reverse bend



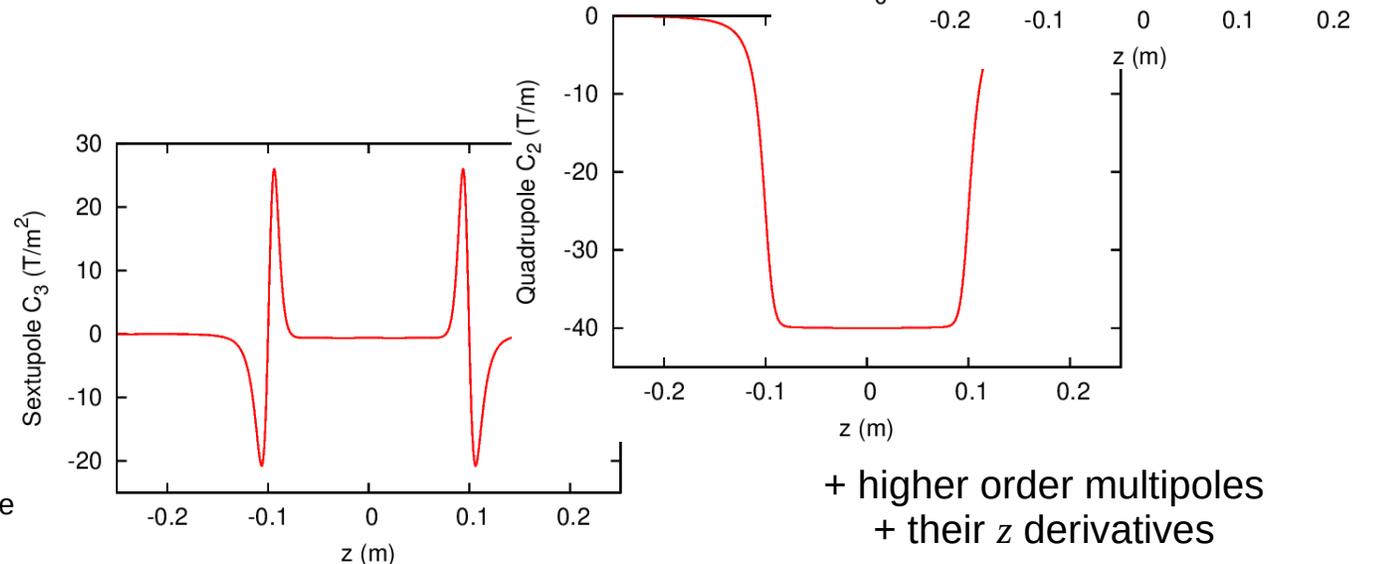
[9] L. Farvacque *et al.*, IPAC 2013, pp 79; L. Farvacque, *et al.*, “ESRF-EBS Design Report,” ed. by D. Einfeld and P. Raimondi (The European Synchrotron, 2018).

[10] M. Borland *et al.*, “Lower Emittance Lattice for the Advanced Photon Source Upgrade Using Reverse Bending Magnets,” in NAPAC 2016, pp. 877

# Generalized gradients from simulated magnetic data

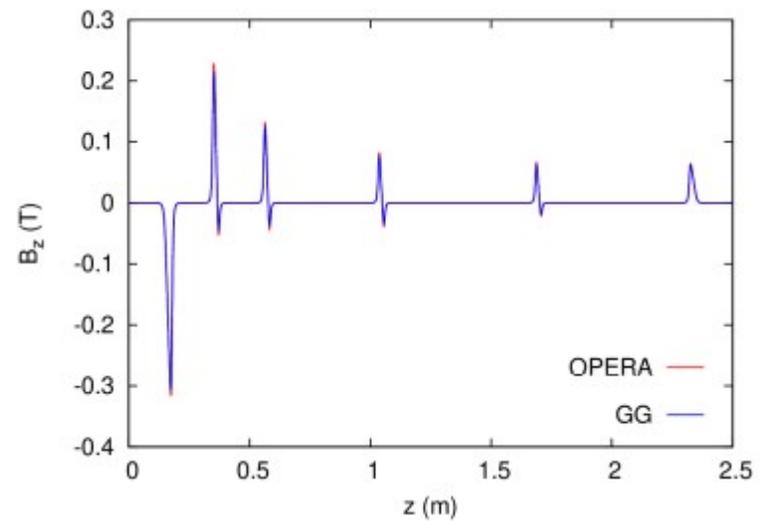
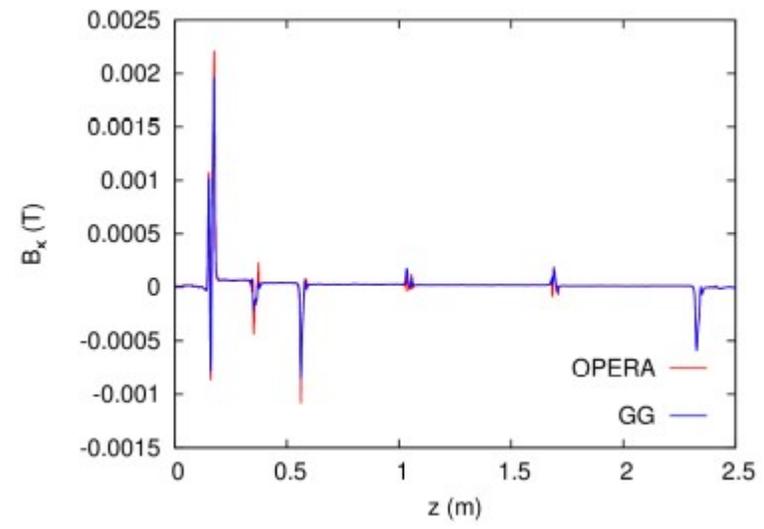
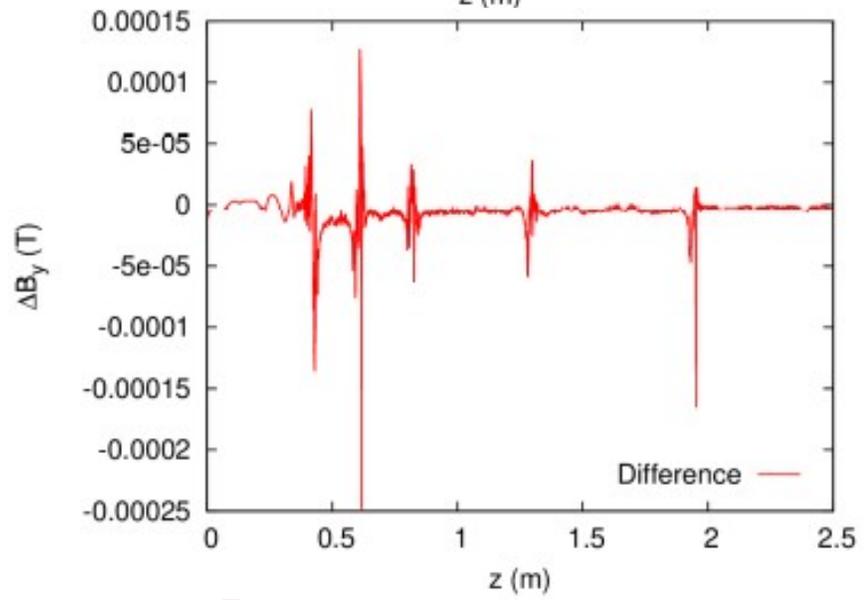
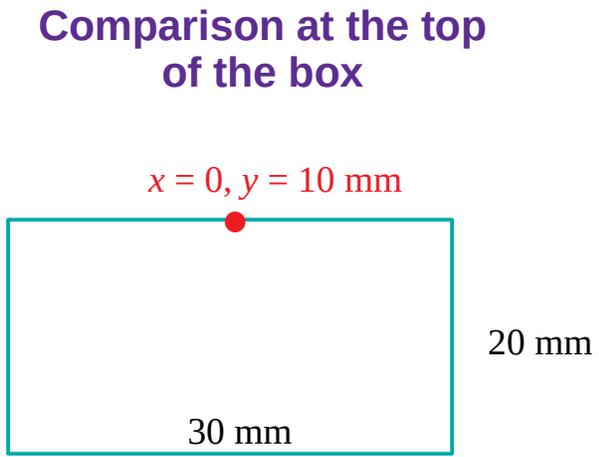
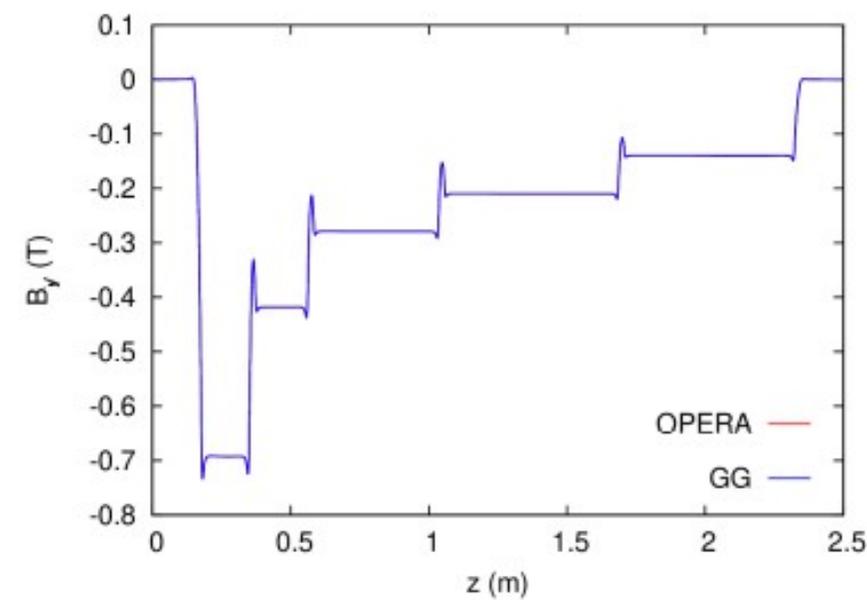
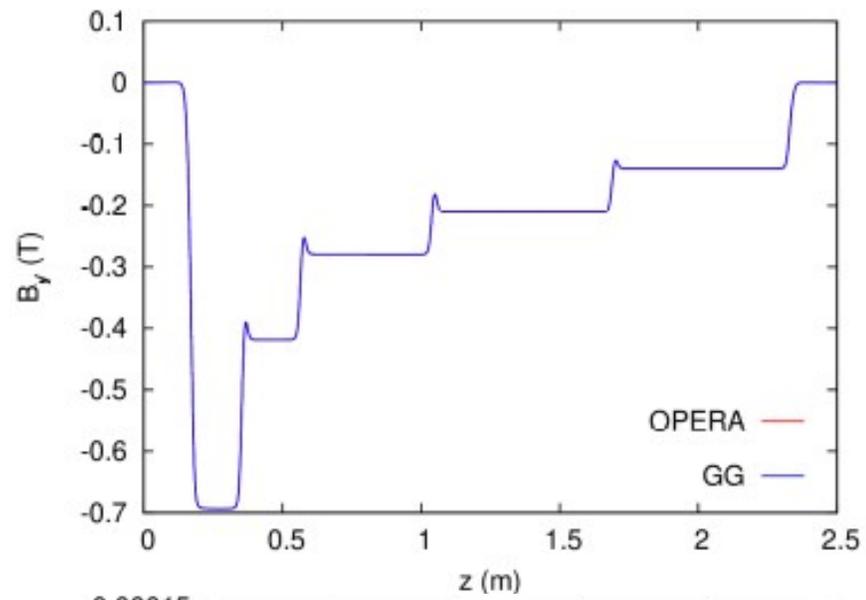


1. Start with simulation data
2. Evaluate normal component of  $\mathbf{B}$  on a bounding surface
3. Compute and retain generalized gradients that minimize  $\Delta\mathbf{B}$  on the boundary
4. Use in tracking



[8] M. Jaski et al. "Magnet Designs for the Multi-bend Achromat Lattice at the Advanced Photon Source," in Proc. IPAC'15, pp. 3260.

# Model of the longitudinal gradient dipole looks good

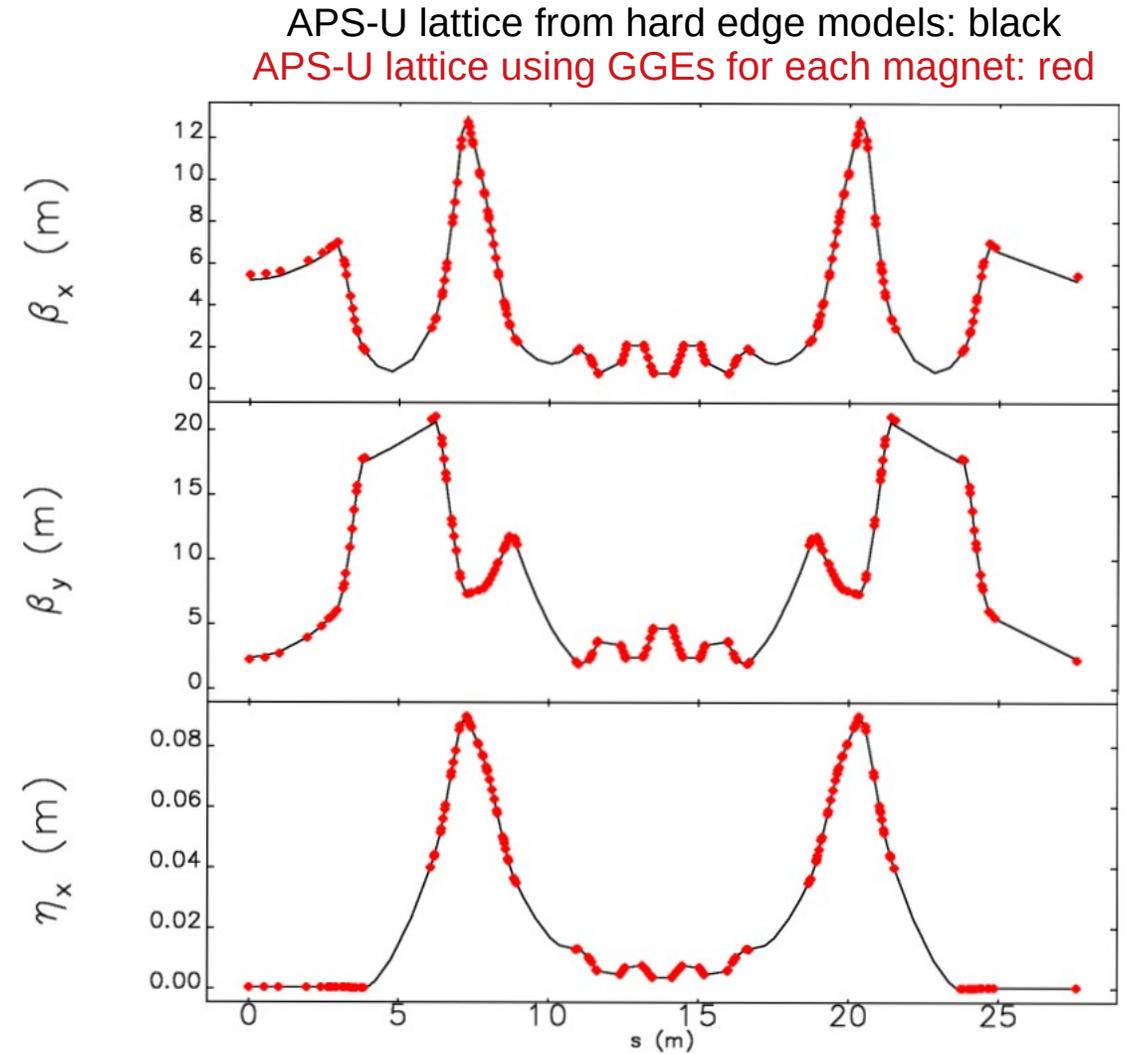


# All-GGE lattice of APS-U tuned to match design

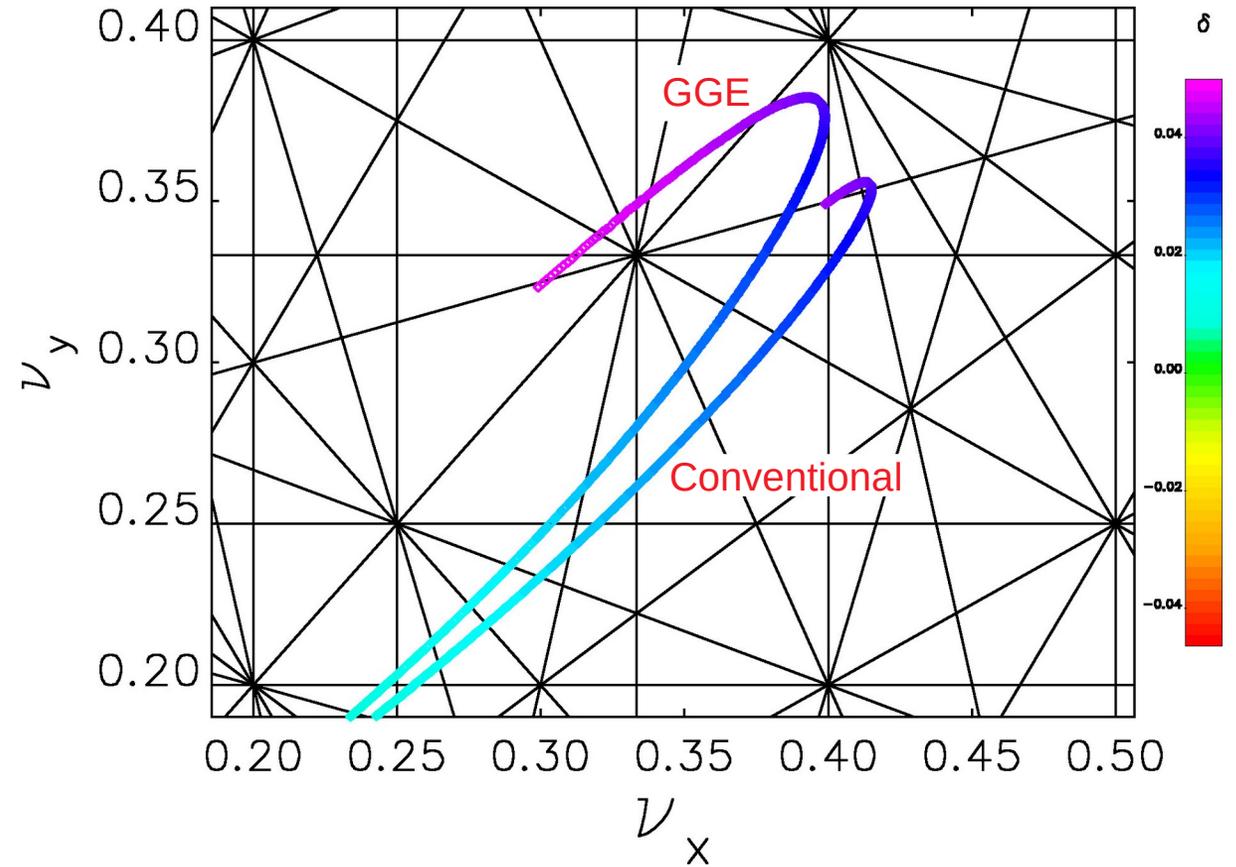
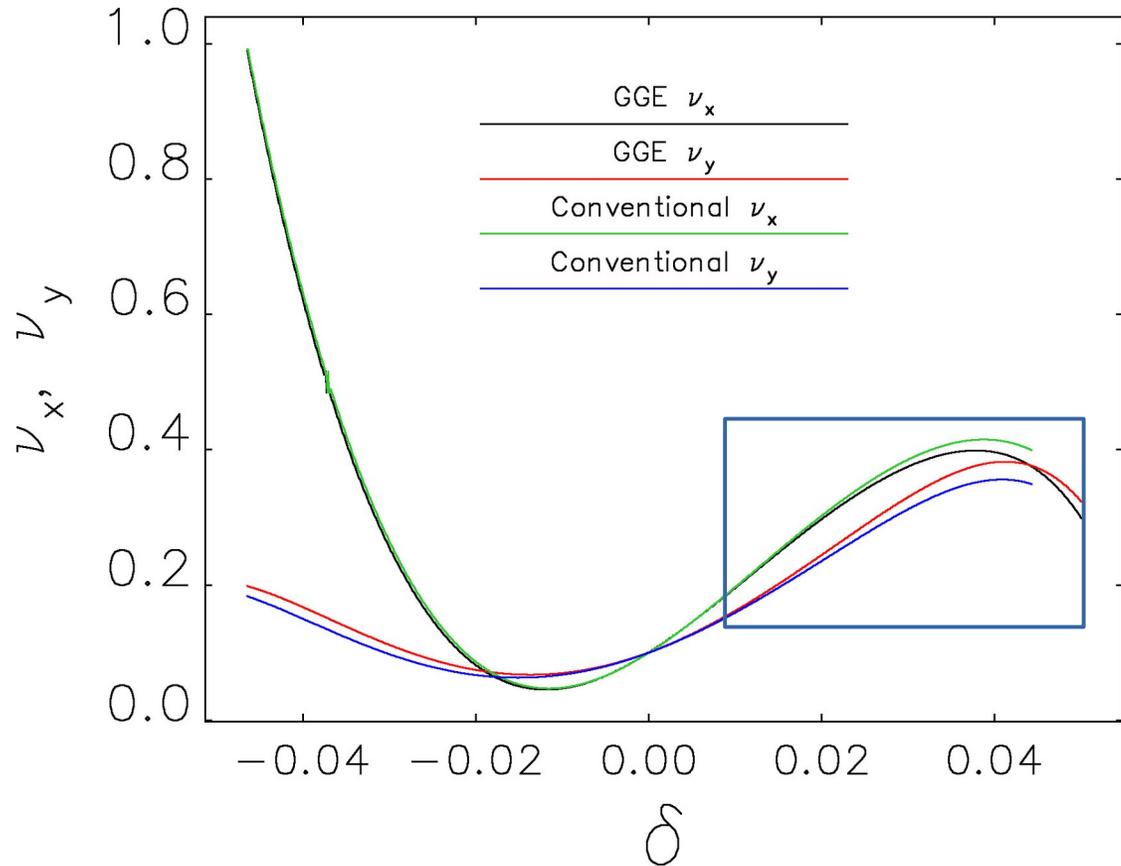
- We used OPERA data from M. Jaski to assemble an all-GGE APS-U lattice model
- Matching of models requires two steps
  - Tune each GGE element to match the 2<sup>nd</sup> order properties of each magnet
  - Apply global tuning to control the orbit and reproduce the linear optics and chromaticity
- This is laborious, but works well
  - Relies on the numerical computation of 2<sup>nd</sup>-order transport matrices<sup>[11]</sup>
  - Optimization is only practical because of parallelization<sup>[12]</sup>

[11] M. Borland, "A High-Brightness thermionic microwave electron gun," PhD thesis, Stanford University, SLAC-402, (1991).

[12] Y. Wang and M. Borland, "Pelegant: A Parallel Accelerator Simulation Code for Electron Generation and Tracking," AIP Conf. Proc., 877, 241 (2006).

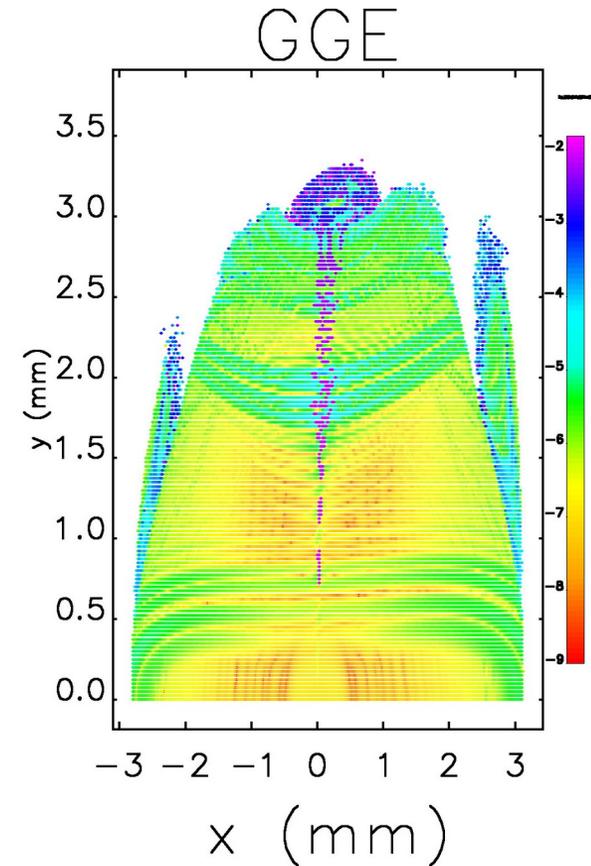
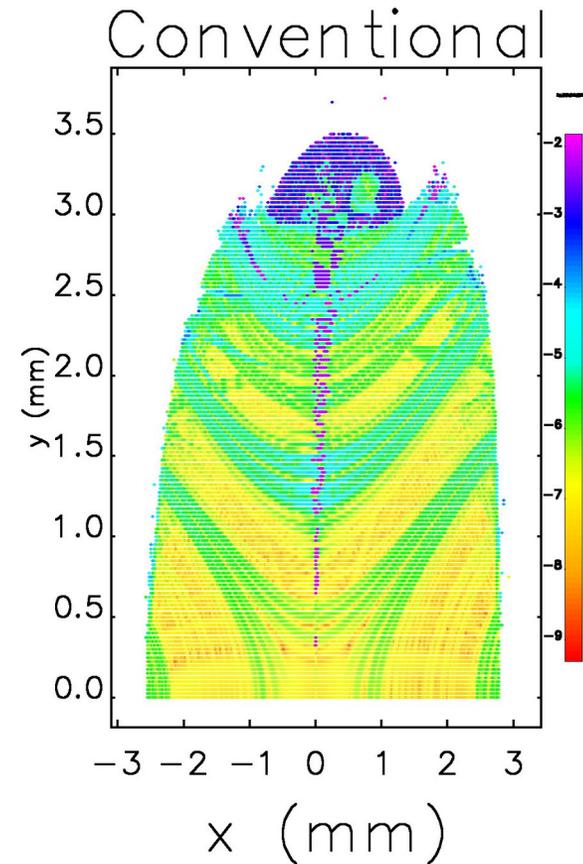
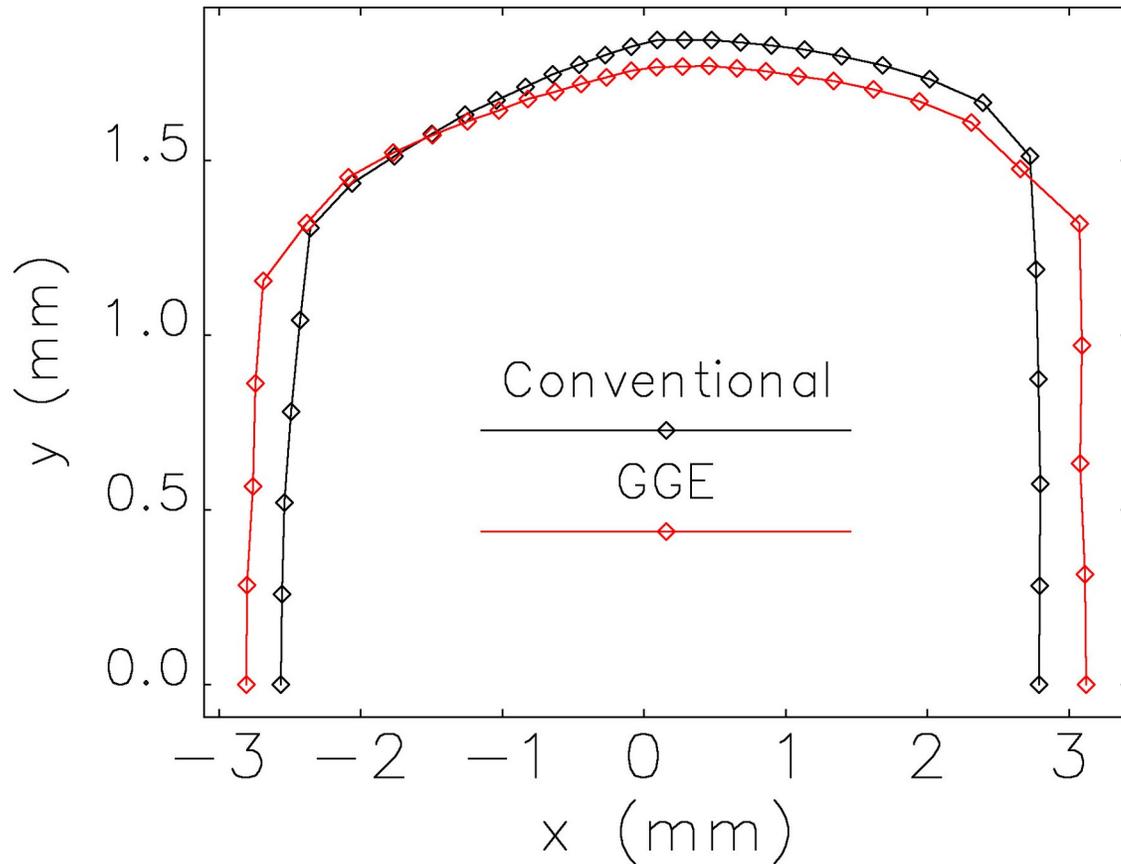


# Chromatic tune footprint matches fairly well



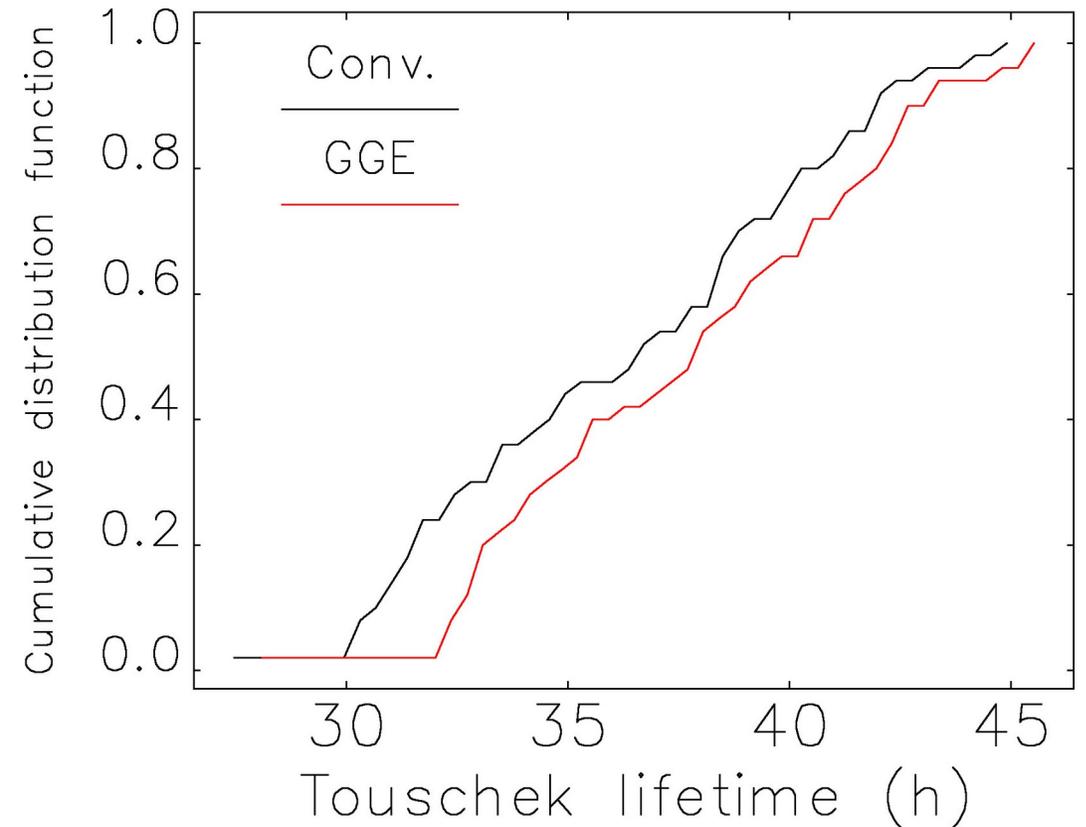
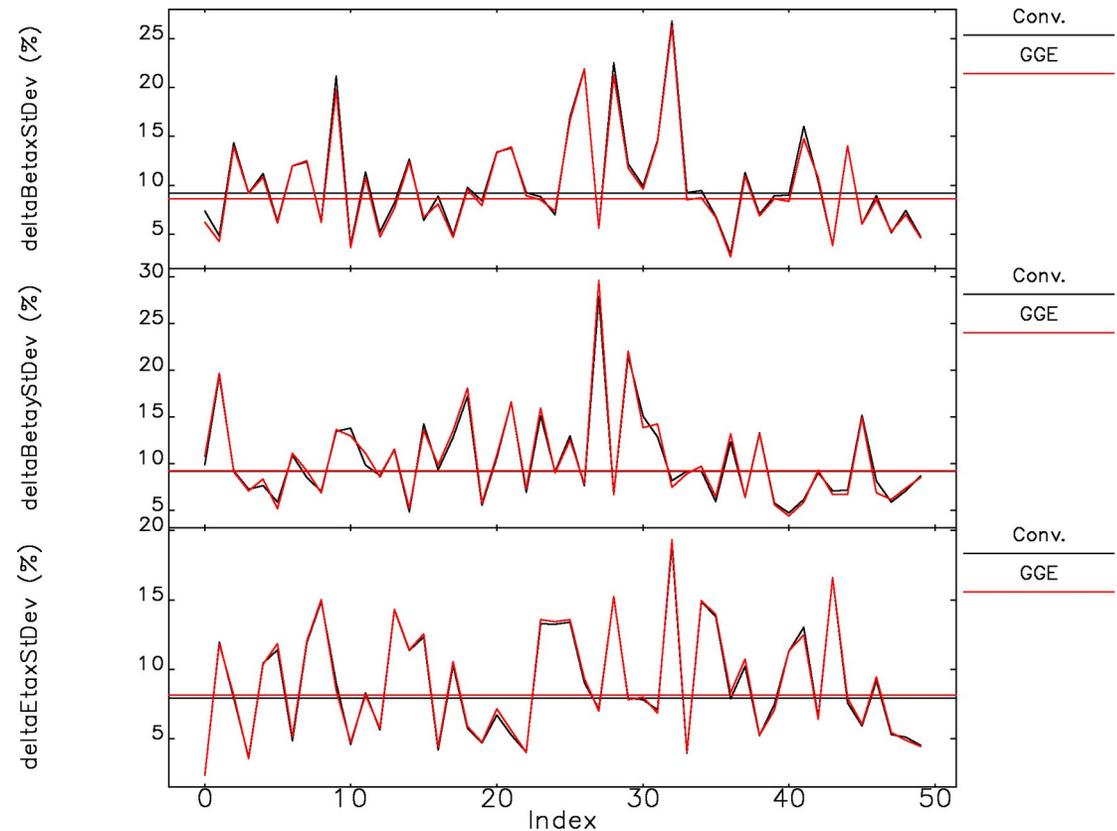
- The tune's dependence on energy is quite close over the entire range
- GGE "tuning" only matched linear optics and chromaticities
- GGE tracking takes about 280 times longer

# Nonlinear dynamics are similar



- The predictions for the dynamic acceptance agree reasonably well
- The frequency maps are vaguely similar
  - Same overall shape, but clearly different details
  - We are investigating possible sources of discrepancy

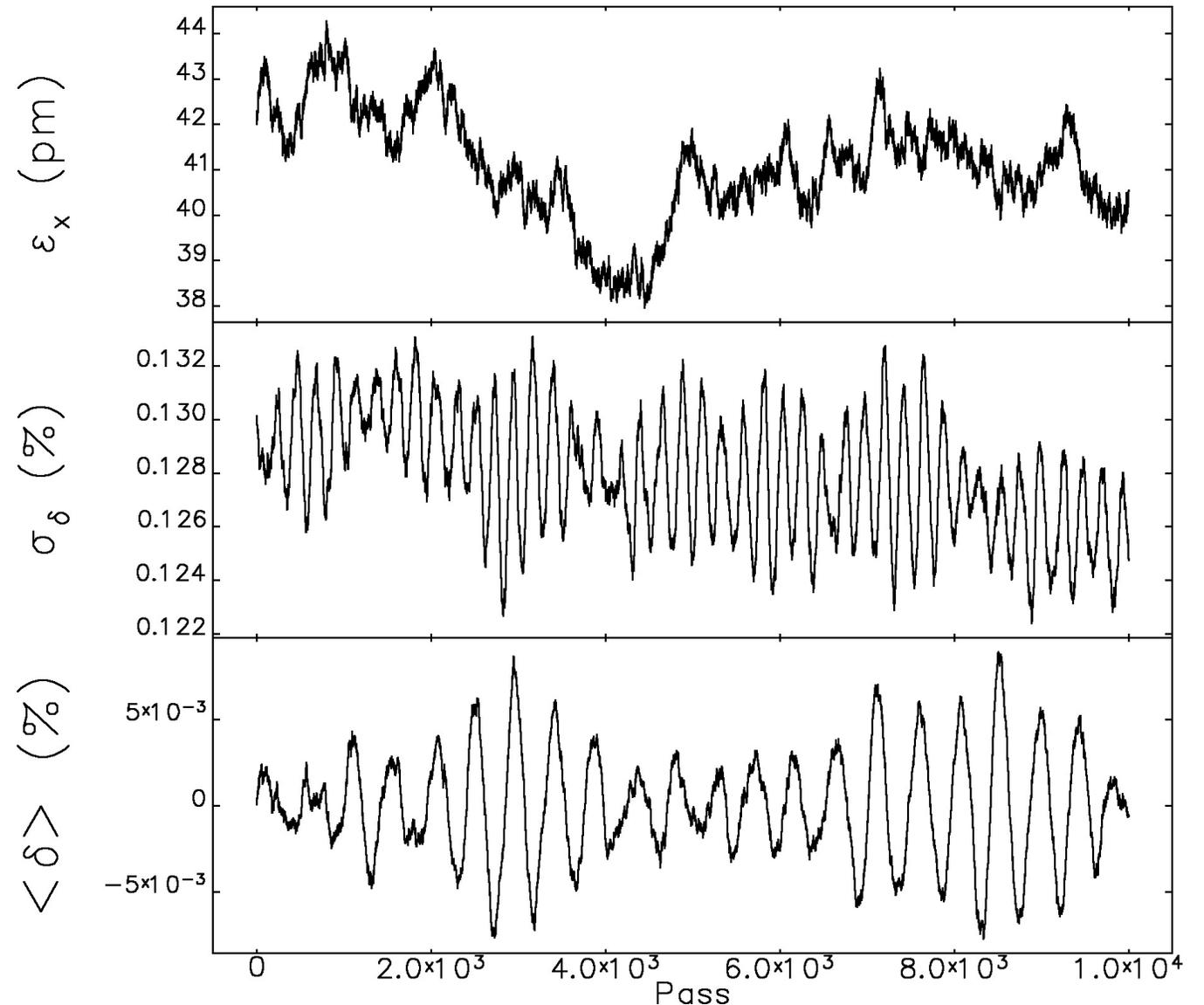
# GGE and usual tracking respond similarly to errors



- Adding 30 micron rms misalignments to all sextupoles results in very similar lattice beating
- Computed the local momentum acceptance using 1000 turns for 50 instances of each case
- Resulting Touschek lifetimes differ by less than 8%, assuming  $\epsilon_x = \epsilon_y = 30$  pm,  $\sigma_\delta = 0.12\%$ ,  $\sigma_t = 100$  ps
  - Note: direct tracking using large sextupole offsets to model errors would result in a larger emittance

# GGE model confirms APS-U emittance

- Ultra-low emittance is a key deliverable for the APS-U
- The implementation of BGGEXP includes synchrotron emission
- Tracking of 1,000 particles, averaged over 5,000 turns:
  - Emittance = 41.0 pm ☺
  - Energy spread = 0.127% ☺
  - 48,000 core hours (!)
- Diffusion matrix computation takes ~200 core hours, and gives essentially identical results



# The magnitude of the GGE tuning indicates that some hard-edge models could be improved

- After tuning the GGE model, the straight magnets have integrated strengths very close to design values
- Matching the transverse gradient dipoles require changing the GGE dipole and quadrupole strengths by a few percent
- Matching the longitudinal gradient dipoles requires small strength adjustments, but large (~2 mm) longitudinal displacements.
  - Hard edge model of longitudinal gradient dipole has long been troublesome
  - Could we improve matters with better fringe field modeling?

## Comparison of integrated strength

Magnet Name	Design to GGE ratio	Design length (m)	GGE length (m)
Q1	1.0107	0.20495	0.20492
Q2	1.0002	0.17918	0.17916
Q3	0.9979	0.18009	0.18005
Q6	1.0010	0.18010	0.17974
Q7	0.9967	0.35655	0.35655
S01A:S1	0.9924	0.18050	0.18046
S01A:S2	0.9892	0.21075	0.21056
S01A:S3	0.9924	0.18050	0.18046
S01B:S1	0.9924	0.18050	0.18046
S01B:S2	0.9892	0.21075	0.21056
S01B:S3	0.9924	0.18050	0.18046
S02A:S1	0.9924	0.18050	0.18046
S02A:S2	0.9892	0.21075	0.21056
S02A:S3	0.9924	0.18050	0.18046
S02B:S1	0.9924	0.18050	0.18046
S02B:S2	0.9892	0.21075	0.21056
S02B:S3	0.9924	0.18050	0.18046

## Tuning parameters for transverse gradient dipoles

Element Name	Dipole Factor	Quadrupole Factor	DX (mm)
Q4	1.0171	1.0028	-0.024
Q5	0.9681	0.9948	0.009
M3	0.9842	1.0017	0.012
Q8	1.0138	1.0118	-0.021
M4	0.9833	1.0117	-0.077

## Tuning parameters for longitudinal gradient dipoles

Element Name	Dipole Factor	DZ (mm)
AM1	0.9999	-1.376
AM2	0.9987	1.999
BM2	0.9987	-1.999
BM1	0.9999	1.376

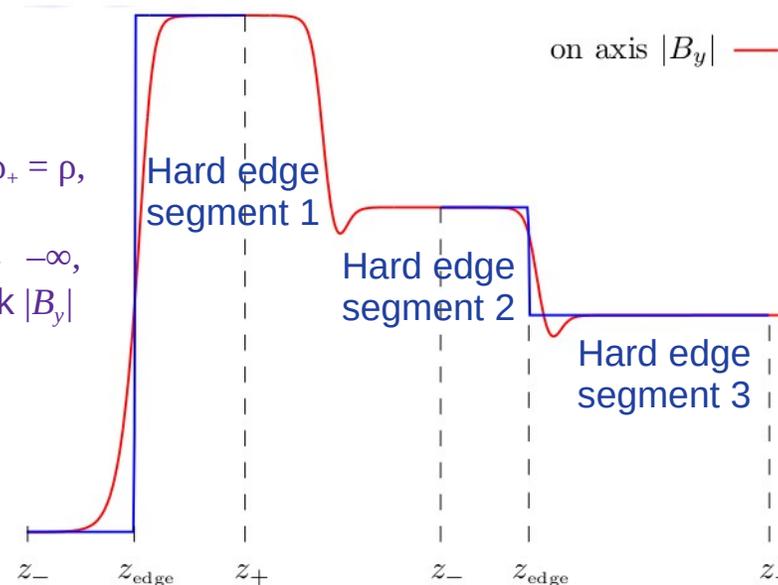
# A hard edge model of more complicated magnets

- The field in the hard edge magnet only depends on the transverse coordinates
  - Fields have unambiguous description in terms of multipole components
  - Tracking with explicit, symplectic integrators is possible using splitting methods
- The difference between the dynamics within the hard edge model and that in the actual magnetic field is collected under the umbrella of “fringe field” effects
- We define the hard edges such that the integrated bending field of the model matches the real magnet:

$$\int_{z_-}^{z_+} dz B_y(0, 0, z) = \int_{z_-}^{z_+} dz C_1(z) = (z_+ - z_{\text{edge}})C_1(z_+) + (z_{\text{edge}} - z_-)C_1(z_-) = (z_+ - z_{\text{edge}})\frac{p_0}{q} \frac{1}{\rho_+} + (z_{\text{edge}} - z_-)\frac{p_0}{q} \frac{1}{\rho_-}$$

## Magnet entrance:

- Bending radii  $\rho_+ = \rho$ , while  $\rho_- \rightarrow \infty$
- Endpoints  $z_- \rightarrow -\infty$ , while  $z_+$  at peak  $|B_y|$



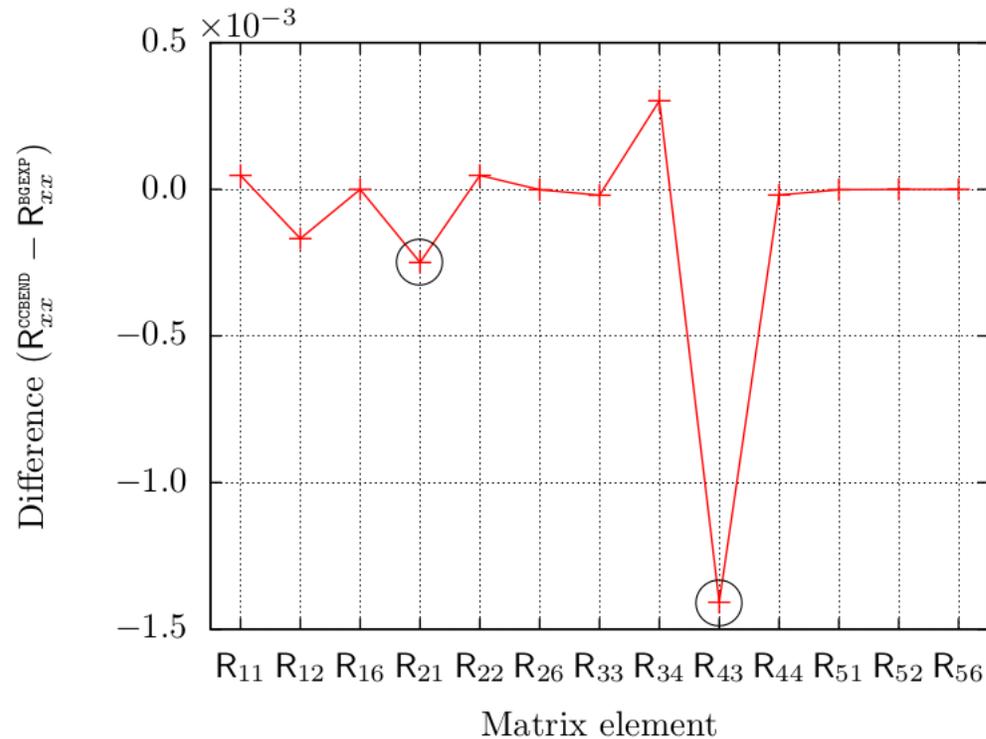
Multipole content is defined by the on-axis gradient + hard edges

## Fringe between segments:

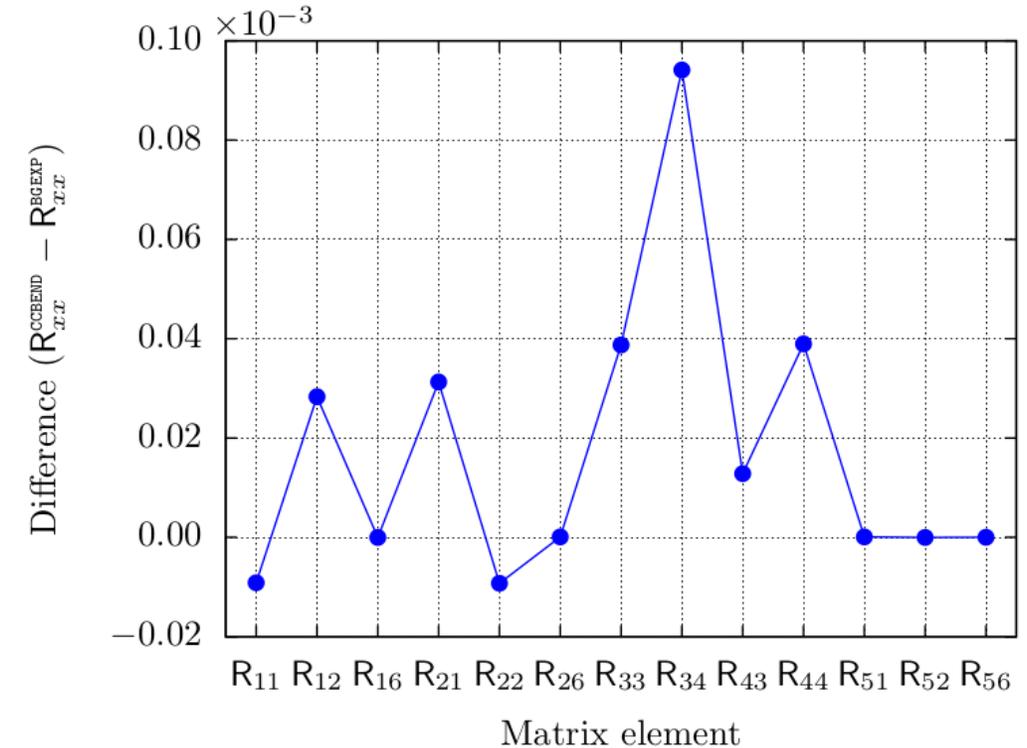
- Non-zero bending radii  $\rho_+$  and  $\rho_-$  both upstream and downstream
- Endpoints  $z_{\pm}$  near the center of the flat-field region
- Fringe field is “more interesting”

# Linear matrix element comparisons using the improved fringe field model for the Q4

CCBEND model<sup>[13]</sup> but with no fringe effects



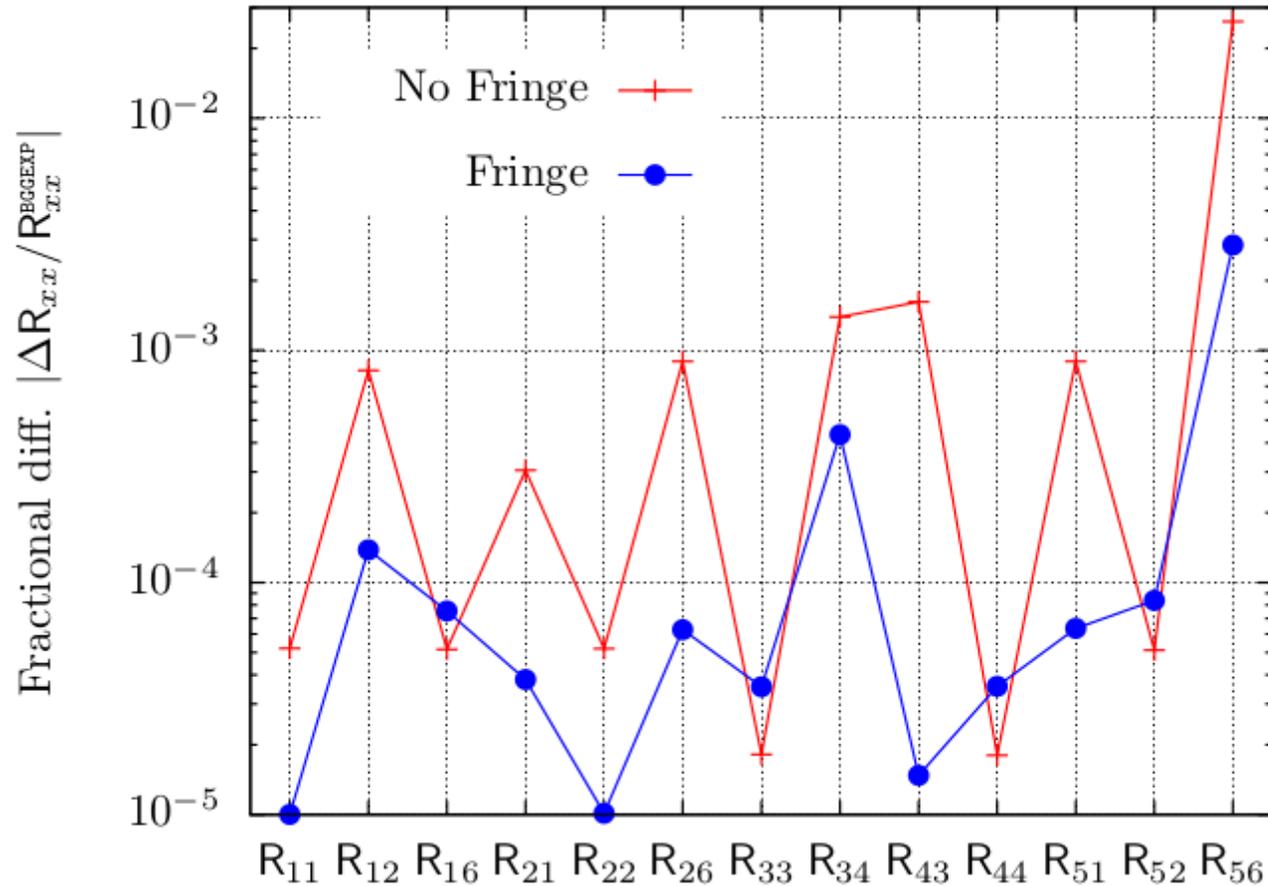
“Complete” CCBEND model with fringe effects



- Linear matrix elements of the hard edge model differ from GGE tracking by < 0.15%
- Improved fringe theory reduces differences in the linear matrix elements to the few  $\times 10^{-5}$  level or better
- Hard edge model has “too much” focusing in both horizontal and vertical planes

[13] M. Borland, “Symplectic Integration in elegant,” Tech. Rep. LS-356, Advanced Photon Source, 2021.

# Improved accuracy of the Q4 fringe field model is required to obtain good tune predictions



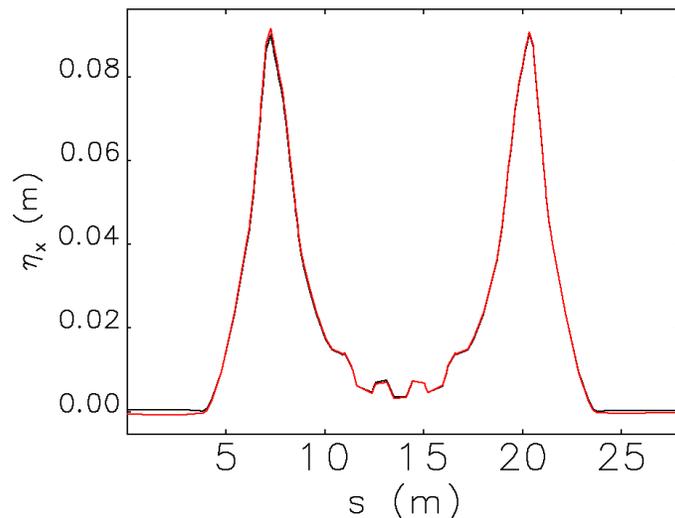
- Fractional error in the linear matrix elements are
  - ~ 0.1% without fringe contributions
  - ~ 0.02% including fringe terms
- Tunes are sensitive to the focusing in Q4 reverse bend due to its large beta function
- Are these models good enough for accurate modeling of the APS-U lattice?
  - Comparison of tunes shows very good agreement with fringe model included

Model	$\nu_x$	$\nu_x$
No Fringe	95.0038	36.1560
Fringe	94.9832	36.0872
BGGEXP	94.9856	36.0878

We feel reasonably confident that we are accurately modeling our transverse gradient reverse bends

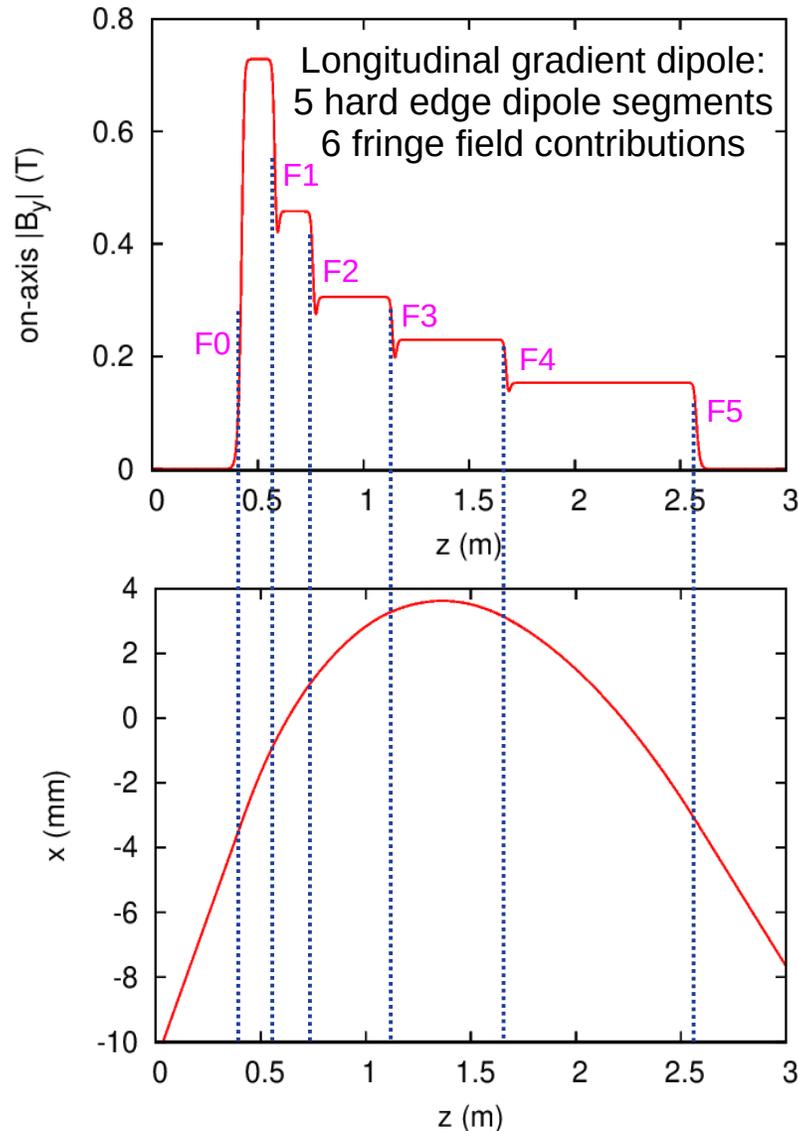
# Very good results are found for both Q4 and Q5

Model	$\beta_x$ (m)	$\beta_y$ (m)	$\eta_x$ (mm)	$\nu_x$	$\nu_y$	$\xi_x$	$\xi_y$
BGGEXP Q5	5.220	2.406	0.3938	95.116	36.076	-133.95	-111.39
CCBEND Q5	5.219	2.406	0.3936	95.115	36.076	-133.94	-111.39
BGGEXP Q4	5.071	2.398	0.3507	94.986	36.088	-131.45	-111.79
CCBEND Q4	5.068	2.399	0.3471	94.983	36.087	-131.41	-111.79
BGGEXP Q4+Q5	5.102	2.413	-0.6282	95.001	36.064	-132.09	-111.55
CCBEND Q4+Q5	5.085	2.414	0.4601	94.998	36.063	-131.68	-111.55

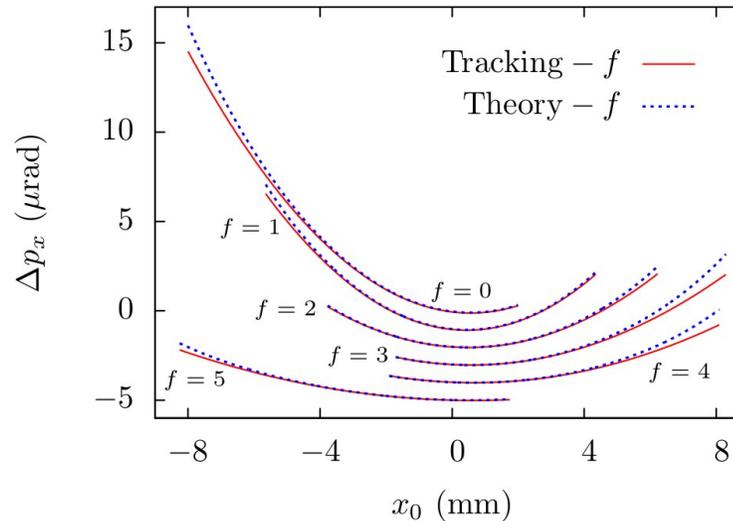


- Tunes agree to within 0.003 for all cases
- For models that just replace the Q4 or Q5 we have
  - Linear lattice function agreement to better than 1%
  - Chromaticities that are essentially identical
- Agreement of lattice functions and chromaticities are somewhat worse when we replace both Q4 & Q5
  - I assume that this is because both are essentially on the integer  $\nu_x$  resonance, but we'll see...

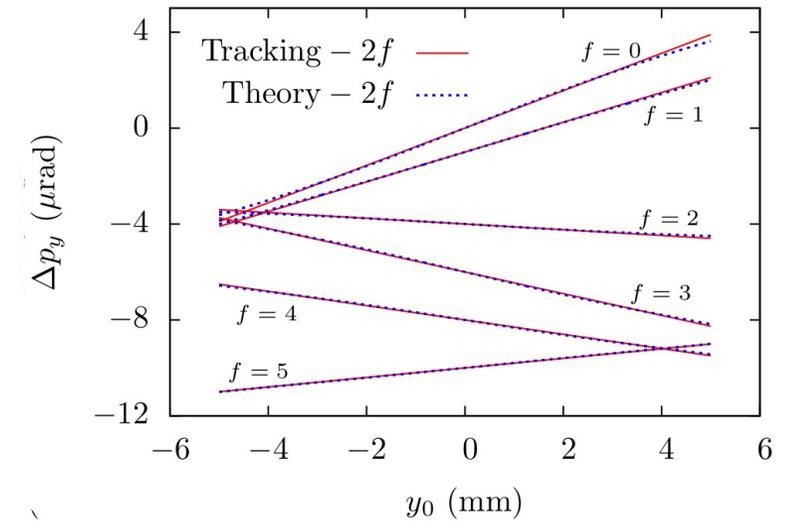
# Generalized gradient tracking has verified a new model of our longitudinal gradient dipole



- Hard edges are set to match integrating bending field
- Fringe field maps defined by (actual field) – (hard edge field)



Linear matrix from generalized gradient tracking (BGGEEXP)

$$\begin{bmatrix} 0.99822 & 2.2243 & 0 & 0 & 0 & 0.04035 \\ -0.00101 & 0.9995 & 0 & 0 & 0 & 0.02857 \\ 0 & 0 & 1.0009 & 2.2251 & 0 & 0 \\ 0 & 0 & 0.00051 & 1.0002 & 0 & 0 \\ 0.02856 & 0.0232 & 0 & 0 & 1 & 0.00029 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$


Linear matrix from the new, hard-edge LGBEND element

$$\begin{bmatrix} 0.99816 & 2.2242 & 0 & 0 & 0 & 0.04029 \\ -0.00106 & 0.9995 & 0 & 0 & 0 & 0.02857 \\ 0 & 0 & 1.0010 & 2.2253 & 0 & 0 \\ 0 & 0 & 0.00056 & 1.0003 & 0 & 0 \\ 0.02856 & 0.0233 & 0 & 0 & 1 & 0.00029 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

# Summary and future directions

- The generalized gradient representation is appealing for accelerator modeling
  - Field is represented as a Taylor series in the transverse coordinates
  - Field is divergence free and suitable for symplectic tracking
- Generalized gradients can be accurately computed from magnetic field data
  - Data can be from simulations or measurements
  - Data can be on rectangular prisms or circular cylinders
- We provide convenient tools to compute the generalized gradients for elegant particle tracking
- We have used generalized gradients for a variety of APS-U modeling tasks
  - Evaluation of effects of leakage fields of a Lambertson septum
  - Verification of nonlinear dynamics and emittance
  - Validation of improved hard edge and fringe models for Cartesian gradient dipoles
- Plans include expanding tools to include curved surfaces
  - Generalized gradient models of dipoles with large bending angles
  - Fringe field modeling for transverse gradient sector bends