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6-D ELEMENT-BY-ELEMENT PARTICLE TRACKING WITH CRAB CAVITY PHASE NOISE AND WEAK-STRONG BEAM-BEAM INTERACTION FOR THE HADRON STORAGE RING OF THE ELECTRON-ION COLLIDER*

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Abstract

The Electron Ion Collider (EIC) presently under construction at Brookhaven National Laboratory will collide polarized high energy electron beams with hadron beams with luminosities up to $1 \times 10^{34}\text{cm}^{-2}\text{s}^{-1}$ in center mass energy range of 20-140 GeV. Crab cavities are used to compensate the geometric luminosity due to a large crossing angle 25 mrad in the EIC. It was found that the phase noise in crab cavities will generate a significant emittance growth for the hadron beams. The phase noise tolerance from the analytical calculation is very small for the Hadron Storage Ring (HSR) of the EIC. In this article, we present our preliminary numerical simulation results with a 6-d symplectic particle tracking to determine the proton emittance growth rate with crab cavity phase noise and beam-beam interaction for the HSR.

INTRODUCTION

The Electron Ion Collider (EIC) presently under construction at Brookhaven National Laboratory will collide polarized high energy electron beams with hadron beams with luminosities up to $1\times10^{34}\text{cm}^{-2}\text{s}^{-1}$ in the center mass energy range of 20-140 GeV. To achieve such a high luminosity, we adopt high bunch intensities for both beams, small transverse beam sizes at the interaction point (IP), a large crossing angle 25 mrad, and a novel strong hadron cooling in the Hadron Storage Ring (HSR) of the EIC [1]. Crab cavities are installed on both sides of IP in each ring to create a local closed horizontal crab dispersion bump to restore head-on collision condition.

For the HSR, the frequency of the main crab cavities is 197 MHz. The maximum crab cavity voltage on either side of IP is about 35 MV, which is provided by 4 cells of 197 MHz crab cavities in two cryomodules. To further straighten the bunch shape, second harmonic crab cavities with a frequency 394 MHz are installed on both sides of IP too. There are 2 cells of 394 MHz crab cavities in one cryomodule on each side, which provide a total voltage about 5 MV.

It has been found that phase noise in crab cavities will generate a significant emittance growth for hadron beams [2]. Experiments were also performed in CERN SPS and confirmed this growth [3]. Analytically, the emittance growth can be calculated with

$$\frac{\mathrm{d}\epsilon_x}{\mathrm{dt}} = \frac{\beta_x^{cc}}{N_{\text{cav}}}\left(\frac{eV_{\text{cav}}f_{\text{rev}}}{2E_b}\right)^2 C_{\Delta\phi}(\sigma_\phi)\frac{2\sigma^2_{\Delta\phi}}{f_{\text{rev}}} \quad (1)$$

where N_{cav} is the number of crab cavities, β_x^{cc} the β function at crab cavities, V_{cav} the voltage per cavity, E_b the beam energy, f_{rev} the revolution frequency, $\sigma^2_{\Delta\phi}$ the total beam sampled phase noise power, $C_{\Delta\phi}$ the crab cavity's phase power density spectrum, $\Delta\phi$ the bunch length measured in RF phase.

For the HSR, to keep a physics store for around 10 hours, we need to have the proton emittance growth less than a few %/hour. Based on Eq. (1), for example, to have a 10%/hour horizontal emittance growth rate, the crab cavity phase noise level $\sigma_{\Delta\phi}$ should be less than 2×10^{-6}rad, which is more than one to two orders in magnitude smaller than that current technology can deliver.

In the following, we will carry out numerical simulation to determine the proton emittance growth with crab cavity phase noise with code SimTrack [4]. The beam and machine parameters for the collision mode involving 275 GeV protons and 10 GeV electrons are used [5]. Instead of using a simplified ring model, we adopt the latest HSR design lattice. The purpose of this study is to verify the analytical estimate of phase noise tolerance and to include the effects of beam-beam interaction, nonlinear magnetic field errors in the interaction region (IR), magnet alignment roll errors, and so on.

SIMULATION SETUP

For the present HSR design lattice, the horizontal betatron phase advance between the crab cavities on both sides of IP is 175 degrees and therefore the horizontal crab dispersion bump is not locally closed. In simulation, we adjust the voltages of both side crab cavities to obtain the design horizontal crab dispersion $dx/dz = 12.5\times10^{-3}$ and $dx'/dz = 0$ at the IP. For this temporary setting, there will be horizontal crab dispersion leakage in the rest of ring.

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Same as the dynamic aperture calculation for the HSR, we install random nonlinear magnetic field errors to all IR magnets. Before particle tracking, we will do orbit correction at IP. The linear chromaticities are set to (2,2). The tunes without beam-beam interaction are set to (28.228, 27.210). For the initial 6-d Gaussian distribution, we intentionally reduce the relative momentum spread and the bunch length from their original design values so that all the macro-particles can be fitted into the central 591 MHz storage RF bucket.

To save the computing time, we first use 3000 macro-particles to represent the proton bunch. Macro-partilce are tracked element-by-element up to 500,000 turns. We calculate the proton emittances and Twiss parameters every 2 turns. After tracking is done, we linearly fit the simulated proton emittances and extrapolate the slope to get a relative emittance growth rate $d\epsilon/dt/\epsilon$ in unit of %/hour.

When beam-beam interaction between the proton and electron bunches are included, weak-strong simulation model is used. The electron bunch is treated as a rigid 6-d Gaussian charge distribution. The rigid electron bunch is divided into 5 longitudinal slices. Each macro-proton will interact with each electron slice in a timed order in each turn. The beam-beam kick to the macro-protons from a 4-d Gaussian electron slice is analytically calculated with the Bassetti-Erskine formula.

As a preliminary study, we first only include 197 MHz crab cavities in simulation. As mentioned above, there are 8 197 MHz crab cavity cells in the collisional IR6 of the HSR. We assume that the phase noises of crab cavity cells are independent. Eight series of phase noise sample data are generated before particle tracking. The settings of phase noises of crab cavities in each turn are the same seen by all macro-particles. Two kinds of crab cavity phase noises are used in our simulation: white and pink noises. In the following, we only present the simulation results with pink noise.

WITHOUT BEAM-BEAM INTERACTION

First we simulate the proton bunch's emittance growth only with pink crab cavity noises. We scanned the RMS value of phase noises from 1×10^{-4} rad down to 1×10^{-6} rad. Figures 1 and 2 show the evolutions of horizontal and vertical emittances in 500,000 turns. The emittances from 1×10^{-4}rad is not shown in these plots since its growth is much faster than other cases. From the plots, the horizontal emittance grows faster with a larger crab cavity noise level. The vertical emittance stays at a same level and shows no growth for all cases, which is predicted by the theory.

Table 1 shows proton bunch's emittance growth rates fitted between 10,000 and 500,000 turns. To quantify the dependence of horizontal emittance growth rate as function of the RMS value of crab cavity phase noise, we fit the simulated growth rate with a function $a\times(\sigma_{\Delta\phi})^m$. We found m is about 1.7 for the simulation data. Based on Eq. (1), the horizontal emittance growth rate is proportional to the square of $\sigma_{\Delta\phi}$.

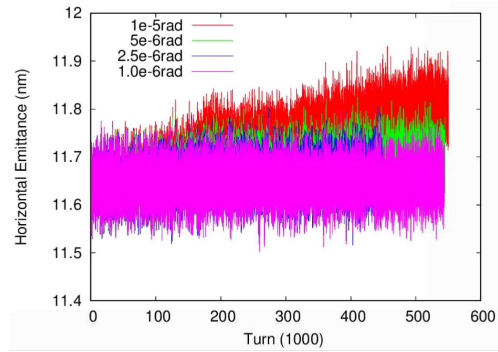

Figure 1: Horizontal emittances with pink crab cavity noises.

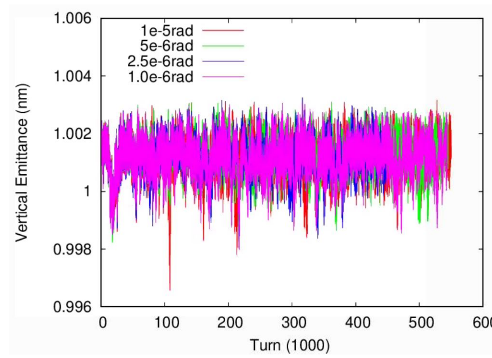

Figure 2: Vertical emittances with pink crab cavities noise.

From Table 1, with 1×10^{-6}rad of crab cavity noise, the simulated horizontal emittance growth rate is about 20%/hour. As mentioned earlier, with 1.4×10^{-6}rad phase noise, its analytical estimate is about 10%/hour. The simulated growth rate is quite close to the analytical estimate. With our current simulation settings, we believe the uncertainty in the simulated emittance growth rate should be about 10%/hour.

WITH BEAM-BEAM INTERACTION

Next we will include beam-beam interaction in the numerical simulation with crab cavity pink phase noise. Figures 3 and 4 show the horizontal and vertical emittances in 500,000 turns. Table 2 shows the simulated proton's emittance growth rates fitted between 10,000 to 500,000 turns.

Compared to the simulation results without beam-beam interaction, there are not much difference in the horizontal emittance growth with beam-beam interaction. However,

Table 1: Simulated Proton Emittance Growth Rates with Pink Crab Cavity Phase Noises

Noise level (µrad)	Horiz. beam size growth rate (%/h)	Vert. beam size growth rate (%/h)
10	7.54e+02	1.05e+01
5	2.15e+02	4.58e+00
2.5	9.35e+01	5.27e-01
1	2.02e+01	2.63e+00

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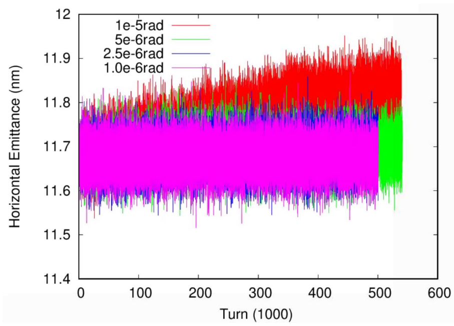

Figure 3: Horizontal emittances with pink crab cavity noise and beam-beam interaction.

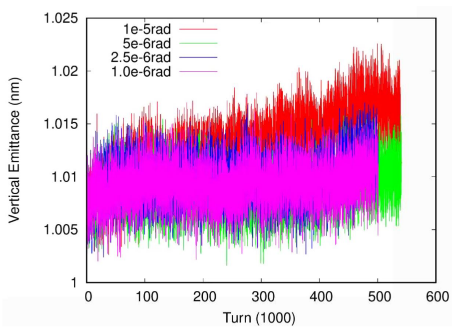

Figure 4: Vertical emittances with pink crab cavity noise and beam-beam interaction.

for the vertical emittance, we can virtually see there is a vertical emittance growth in 500,000 turns with crab cavity phase noises 1×10^{-4}rad and 1×10^{-5}rad. We are looking into the reasons for the vertical emittance growth with both crab cavity phase noise and beam-beam interaction. One possible reason is that nonlinear beam-beam force of a flat beam couples the horizontal emittance growth to the vertical plane. It is also found the vertical emittance growth rate is slower with a rounder transverse beam shape at IP.

From Fig. 4, there is no big difference in the vertical emittance evolution with crab cavity phase noises 5×10^{-6}rad, 2.5×10^{-6}rad, and 1×10^{-6}rad. The vertical emittance growth rate fitted between 10,000 to 500,000 turns are about 59-86%/hour. For a cross-check, we did a contrast simulation only with beam-beam interaction and without crab cavity phase noise, the simulated vertical emittance growth rate is about 53%/hour. There are several issues we are looking into the causes of vertical emittance growth only with beam-beam interaction in this study, such as the closure of crab dispersion bump, without 394 MHz crab cavities in simulation, and a shorter tracking turns than a simplified long-term weak-strong simulation.

WITH MAGNET ROLL ERRORS

We also start to look into the proton bunch's vertical emittance growth coupled from the horizontal emittance growth due to residual betatron coupling. Any coupling sources in the HSR, such as alignment roll errors of quadrupoles, uncorrected vertical closed orbits in sextupoles, and the roll errors

Table 2: Simulated Proton Emittance Growth Rates with Pink Crab Cavity Phase Noise and Beam-beam Interaction

Noise level (μrad)	**Horiz. beam size growth rate (%/h)**	**Vertical beam size growth rate (%/h)**
10	6.59e+02	4.19e+02
5	2.12e+01	7.27e+01
2.5	4.93e+01	8.55e+01
1	-7.58e+00	5.90e+01

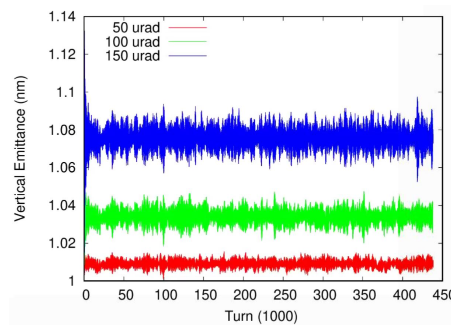

Figure 5: Vertical emittances with random roll errors in all quadrupoles. 2.5×10^{-6} rad crab cavity phase noise is applied.

of crab cavities, will generate a residual global coupling. As a quick check, we carried out simulations with random roll errors to all quadrupoles. The RMS crab cavity phase error 2.5×10^{-6} rad is used for this study. Figure 5 shows the proton beam's vertical emittances with three different random roll errors. From it, the average vertical emittance has changed due to betatron coupling. We do not see much difference in the vertical emittance growth rates among those three cases. The reason may be linked to a small horizontal emittance growth rate with 2.5×10^{-6} rad crab cavity phase noise in this study.

CONCLUSION

In this article, we presented the preliminary simulation results with crab cavity phase noise in the Hadron Storage Ring of the EIC. 6-d symplectic particle tracking and weak-strong beam-beam interaction have been used. Without beam-beam interaction, the simulated horizontal growth rate agree well with the analytical estimate. There is no vertical emittance growth without beam-beam interaction. With beam-beam interaction, we observed vertical growth with 1×10^{-4}rad and 1×10^{-5}rad phase noises. we are not sure if there is vertical emittance growth with a lower phase noise. We will continue this study to understand the observed vertical emittance growth with beam-beam interaction and do more simulations with updated HSR design lattice.

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