

Non-linear Optics from Off-Energy Closed Orbits

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Overview

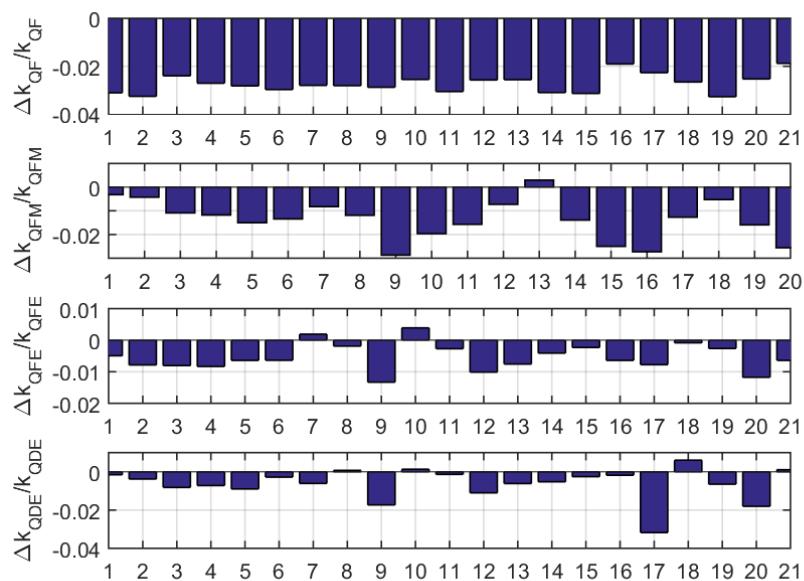
- Motivation
- MAX IV 3 GeV storage ring optics before NOECO
- Simulation studies and characterisation of model
- Proof-of-principle measurements
- Application to the MAX IV 3 GeV storage ring
- Effect on dynamic aperture
- Summary

Motivation

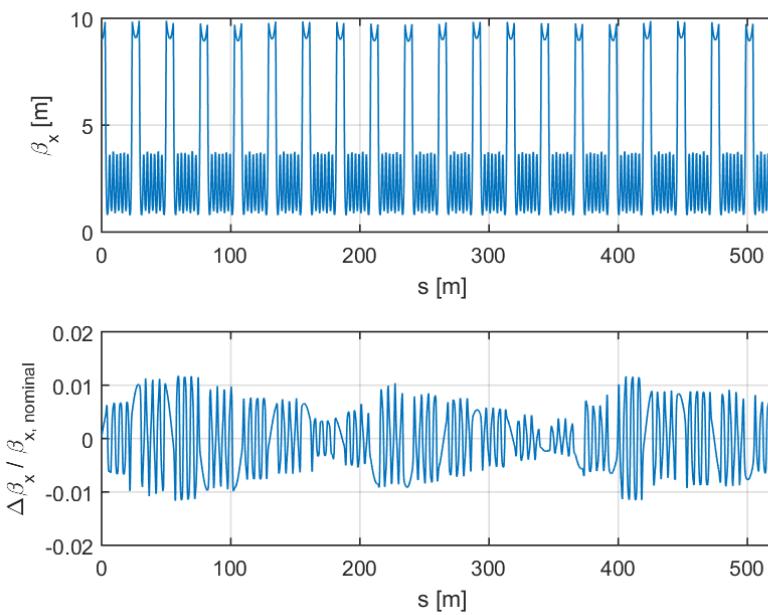
- Increase non-linear optics performance.
 - Injection rate
 - Lifetime
- Find a standardised way to characterise the non-linear optics.
 - Reliable
 - Not too time consuming

Ring Optics before NOECO

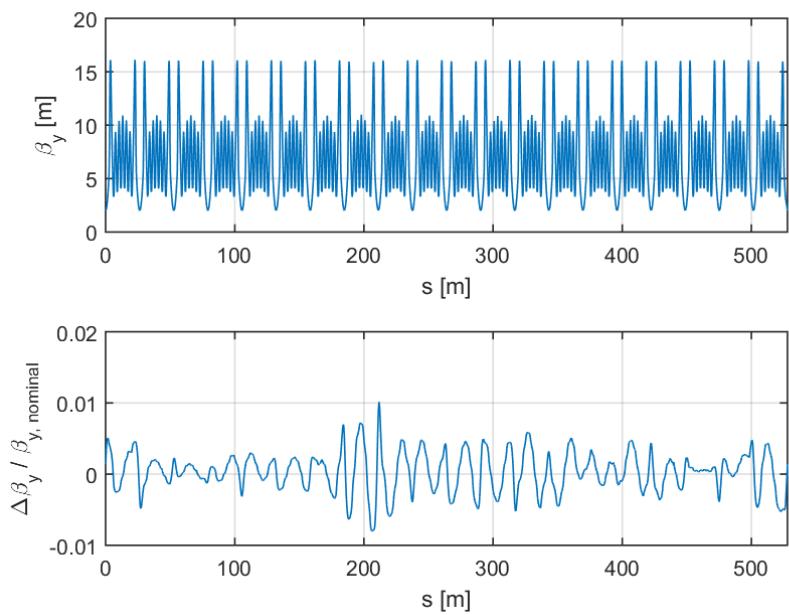
- LOCO regularly used to correct linear optics [1, 2]
 - Correction on the order of $\sim 1.5\%$ relative to magnetic measurements.
 - Corrected beta-beat of $\sim 1 - 2\%$ peak-to-peak.



Relative changes to the quadrupole magnet circuits introduced by LOCO.



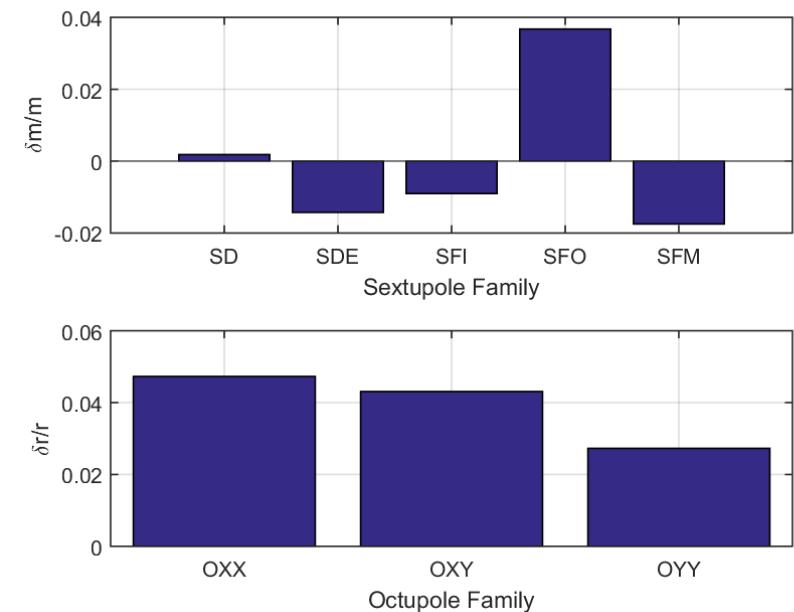
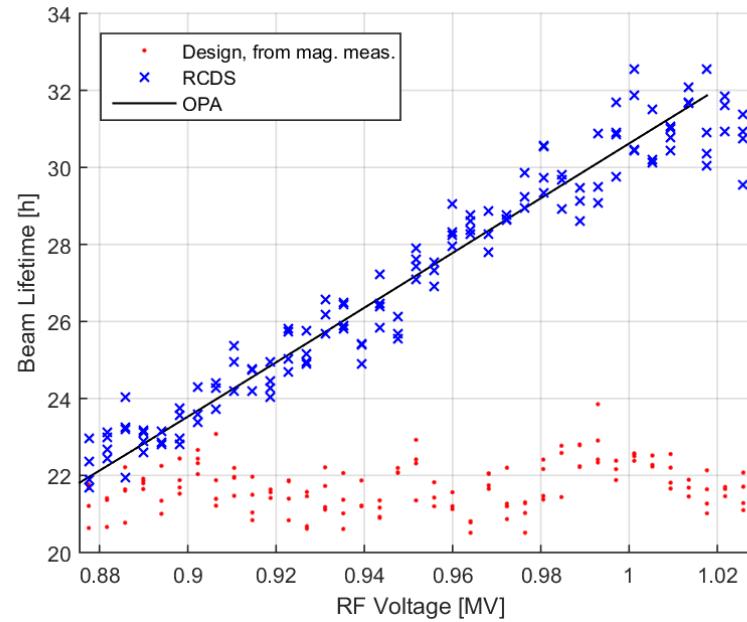
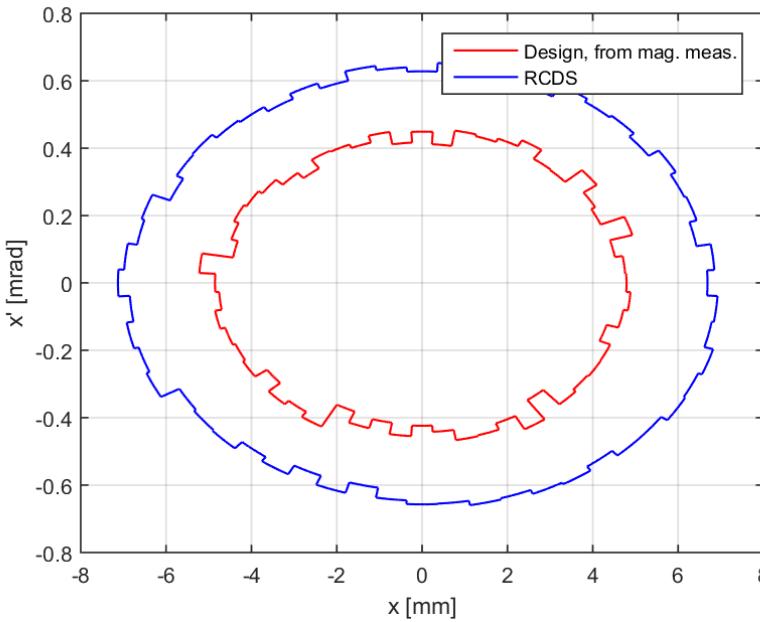
Horizontal beta function and beta-beat after LOCO correction.



Vertical beta function and beta-beat after LOCO correction.

Ring Optics before NOECO

- Sextupole and Octupole family settings optimised for high kick resilience using RCDS [3, 4]
 - Correction on the order of a few percent.
 - Increase of horizontal DA and MA.



Relative changes to the sextupole and octupole magnet families introduced by the RCDS fitting procedure.

Left: DA at centre of straight section measured using the dipole kicker magnets.

Right: Beam lifetime vs. RF voltage at 75 mA stored beam. The pre-RCDS lattice limits the lifetime, while the post-RCDS lattice is limited by RF voltage.

The Chromatic Functions

- Finding a way to characterise and correct non-linear optics to achieve performance predicted by simulations.
- Expand the beta function with regards to energy: $\beta = \beta_0 + \beta_1 \delta + \dots$
- The β_1 terms, the *chromatic functions*, become:

$$\beta_1 = \frac{d\beta(s)}{d\delta} = \frac{\beta_0(s)}{2 \sin 2\pi\nu_0} \oint \beta_0(\sigma) \frac{dp(\sigma)}{d\delta} \cos[2\nu_0(\Psi(s) - \Psi(\sigma) + \pi)] d\sigma$$

$$p_x = (2h^2 + k)\delta - (2h^3 + m + 4hk)\eta\delta - h'\eta'\delta$$

$$p_y = -k\delta + (2hk + m)\eta\delta$$

- Linear dependence on chromatic sextupole strength, m .

The Orbit Response

- Sextupoles could be characterised from the chromatic functions directly.
- The chromatic functions can be extracted from the orbit response matrix at an off-nominal energy (LOCO), but it would result in the loss of phase information.
- We would like to use the response matrix measured at an off-nominal energy to characterise sextupoles, a “non-linear” LOCO.
- Non-linear optics from Off-Energy Closed Orbits – NOECO.

The Off-Energy Orbit Response

$$u(s) = \frac{\sqrt{\beta(s)}}{2 \sin \pi\nu} \sqrt{\beta_\theta} \theta \cos(\pi\nu + \Psi(s) - \Psi_\theta)$$

Inserting the expanded beta function and keeping only order δ and lower

$$\beta = \beta_0 + \beta_1 \delta + \dots$$

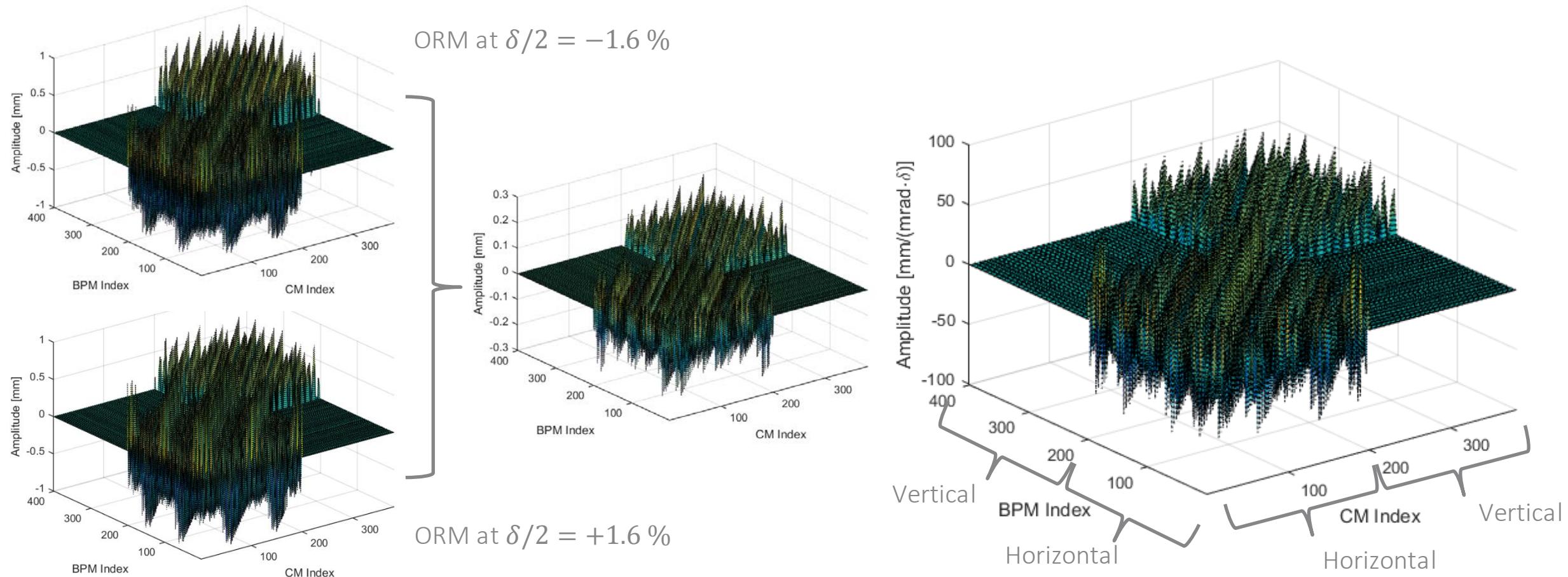
and taking the difference to get only the momentum dependent part:

$$\frac{u_{\delta/2} - u_{-\delta/2}}{\delta} = \frac{1}{4 \sin \pi\nu} \frac{\beta_0 \beta_{1\theta} + \beta_1 \beta_{0\theta}}{\sqrt{\beta_0 \beta_{0\theta}}} \cos(\pi\nu + \Psi(s) - \Psi_\theta)$$

The Off-Energy Orbit Response is approximately linear with chromatic sextupole strengths, m , since β_0 is independent of m . Linearity also holds for a kick at a location with dispersion.

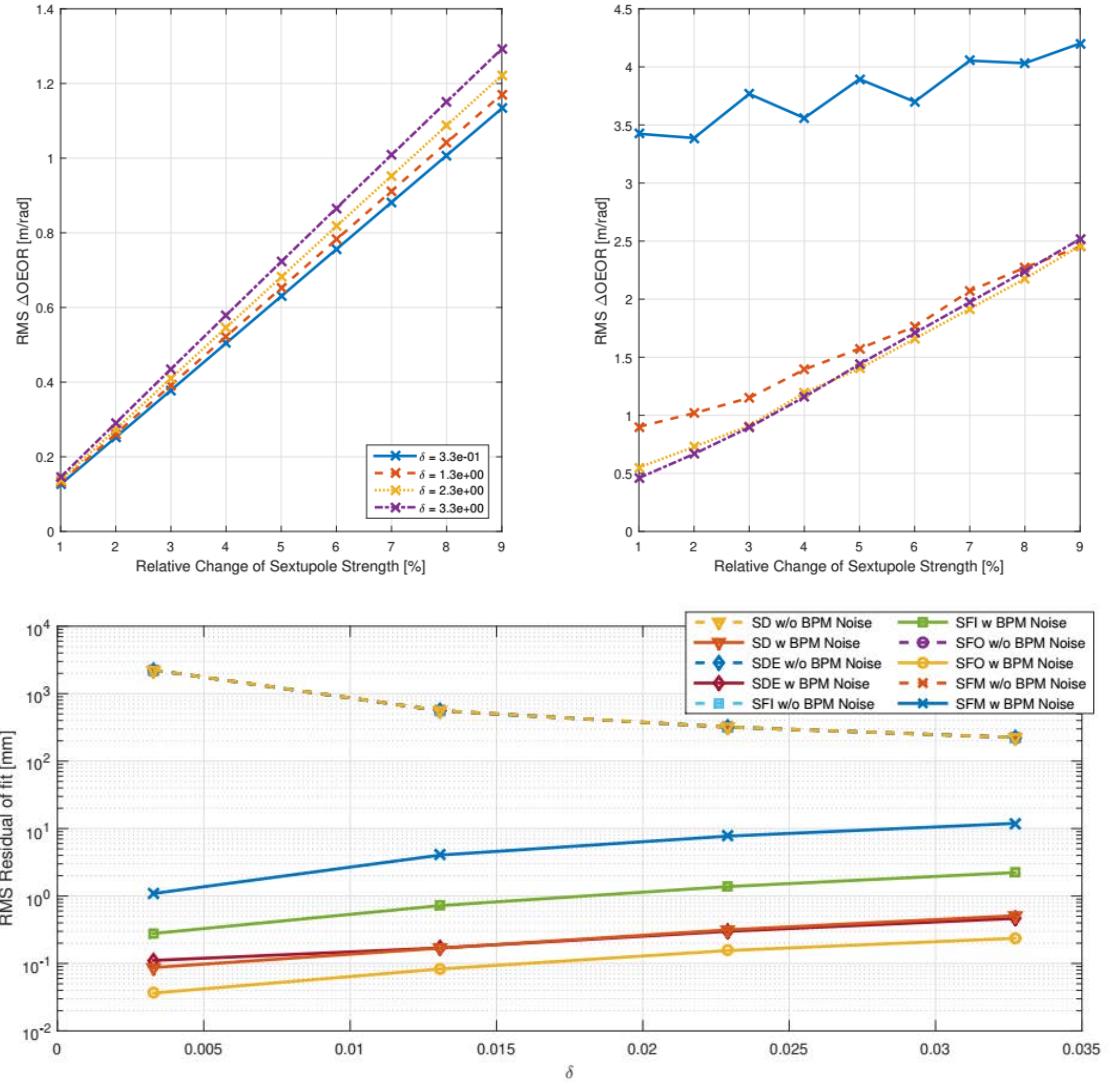
The Off-Energy Orbit Response Matrix

Collecting the off-energy orbit response of all dipole correctors.



Simulation: Choice of Measurement Parameters

- δ and θ determined by looking at the linearity of the OEORM with m at different δ and θ .
- Effect from BPM noise greater than non-linear effect within the achievable range of δ and θ .
- Chosen values:
 $\delta = 3.3\%$
 $\theta = 0.1 \text{ mrad}$ cf. LOCO $\theta \approx 0.013 \text{ mrad}$



Non-linear optics from Off-Energy Closed Orbits

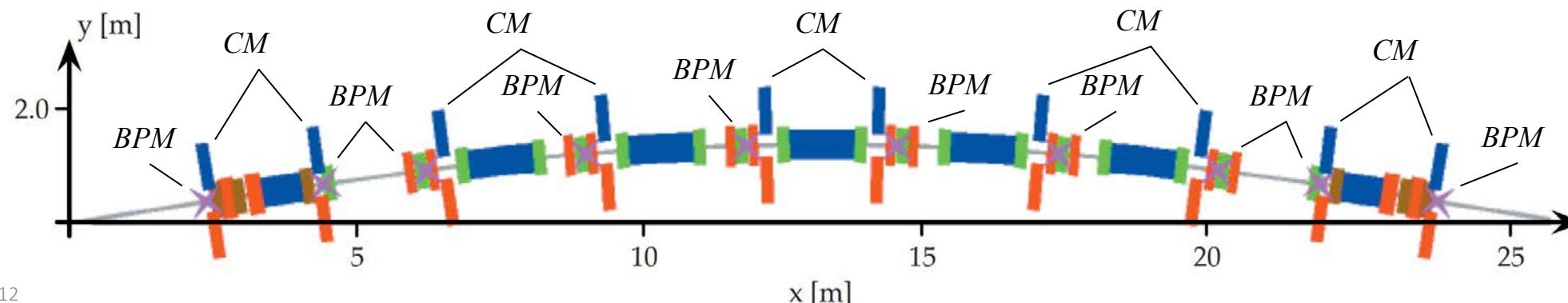
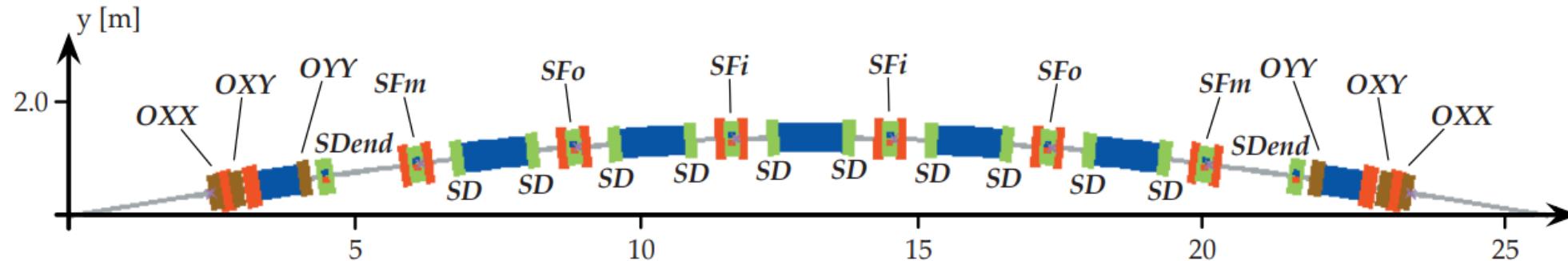
- Minimise the difference between measured and model OEORM .
- Data from each BPM is weighted by its noise level.

$$\begin{aligned}\chi^2 &= \sum_{ij} E_{ij}^2 \\ &= \sum_{ij} \frac{(M_{\text{OEORM,meas},ij} - M_{\text{OEORM,model},ij})^2}{\sigma_i^2}\end{aligned}$$

$$-E_{ij} = \frac{\partial E_{ij}}{\partial K_l} \Delta K_l$$

Choice of Fitting Parameters

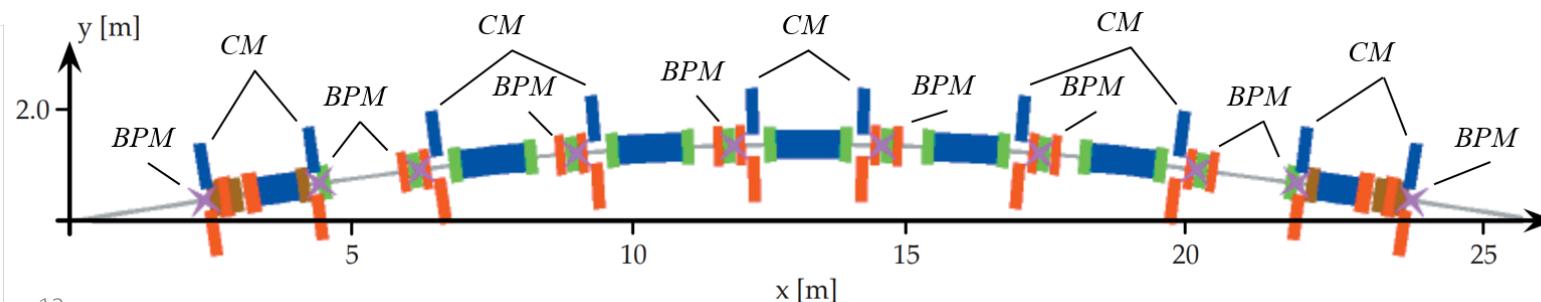
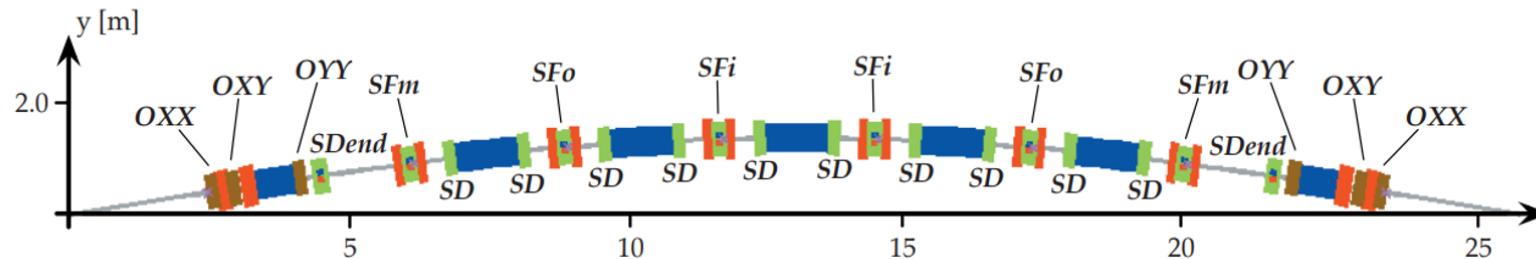
- The OEORM was fitted using the individual sextupole circuits as fitting parameters (one circuit per family and achromat).
- Insufficient sampling for fitting by individual sextupole magnets, nor are the magnets individually powered.



Choice of Fitting Parameters

Fitting parameters:

- 101 individually controlled sextupole circuits \rightarrow 101 fitting parameters.
- Dipole corrector kick strengths \rightarrow 380 fitting parameter.
- 200 BPMs with potential horizontal and vertical gain errors \rightarrow 400 fitting parameters.

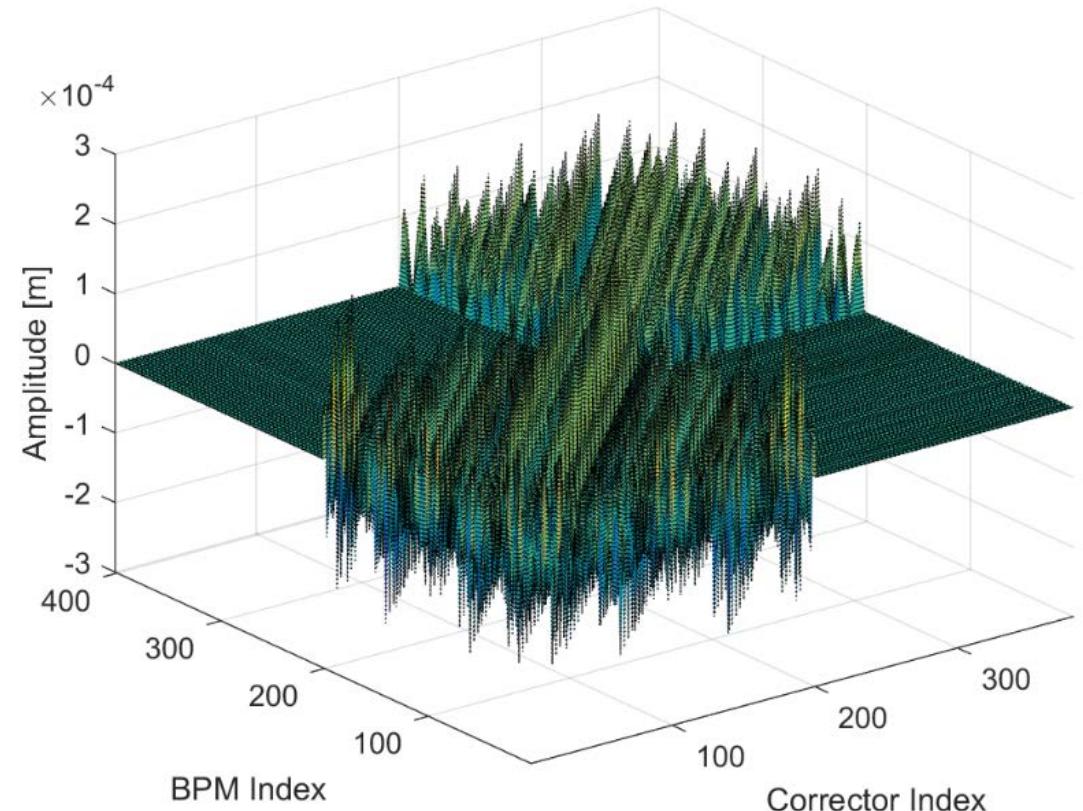


Singularities

- It is not sufficient to have more measurement points than fitting parameters.
 - No guarantee that all fitting parameters can be unambiguously identified from the fitting parameters.

Example:

A uniform increase of the OEORM amplitude can be the result of either BPMs with too high gain, or dipole correctors with too strong kick. Or any combination of the two.

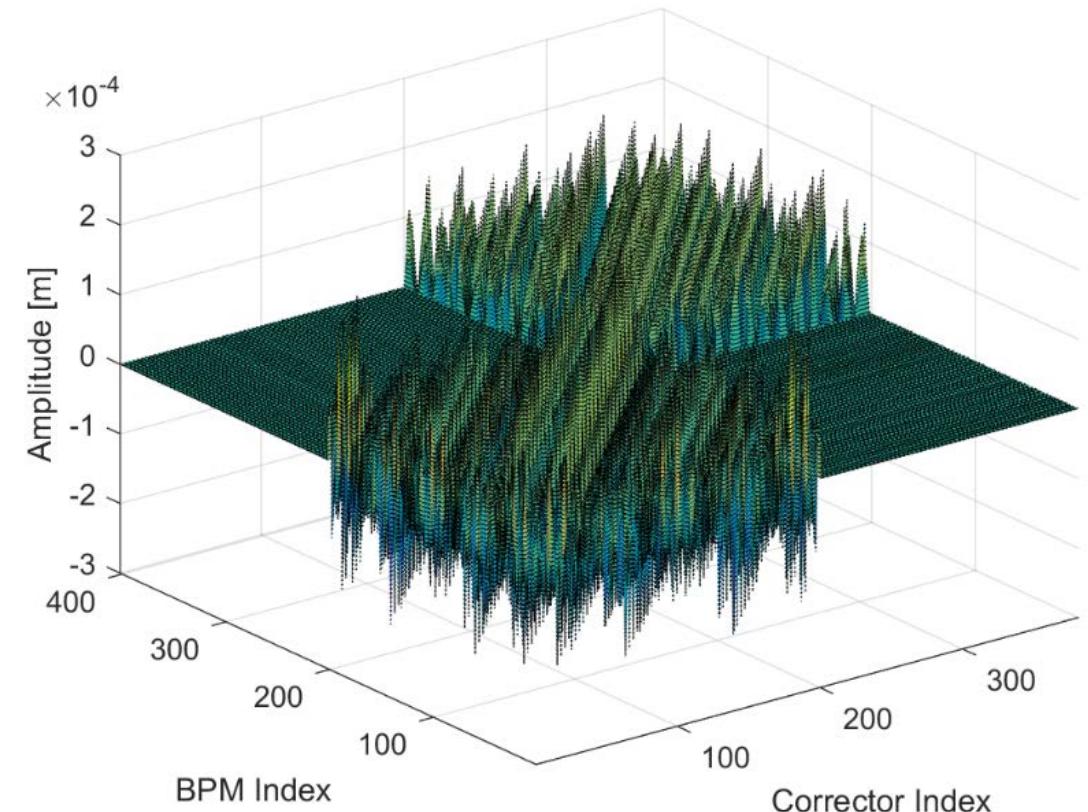


Singularities

Example:

A uniform increase of the OEORM amplitude can be the result of either BPMs with too high gain, or dipole correctors with too strong kick. Or any combination of the two.

- Needs to be resolved for fitting BPM gains and corrector kicks.
- Appending the 2nd order dispersion to the OEORM.

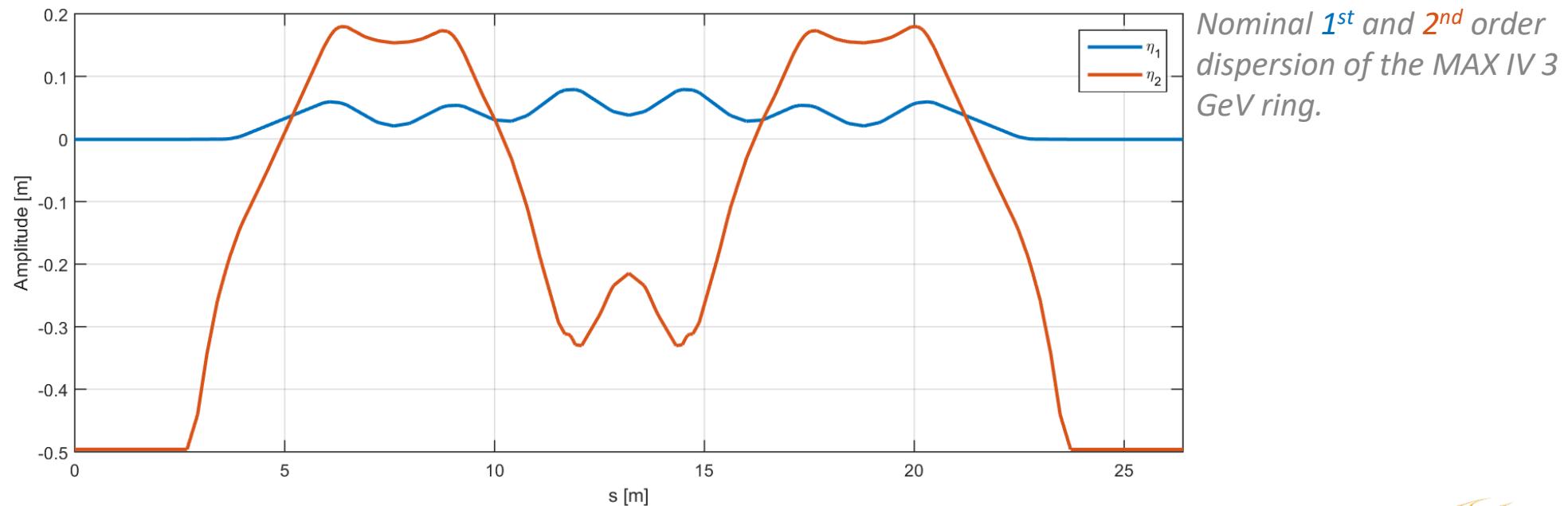


2nd Order Dispersion

- Similarly to the beta function the dispersion can be expanded with regards to energy:

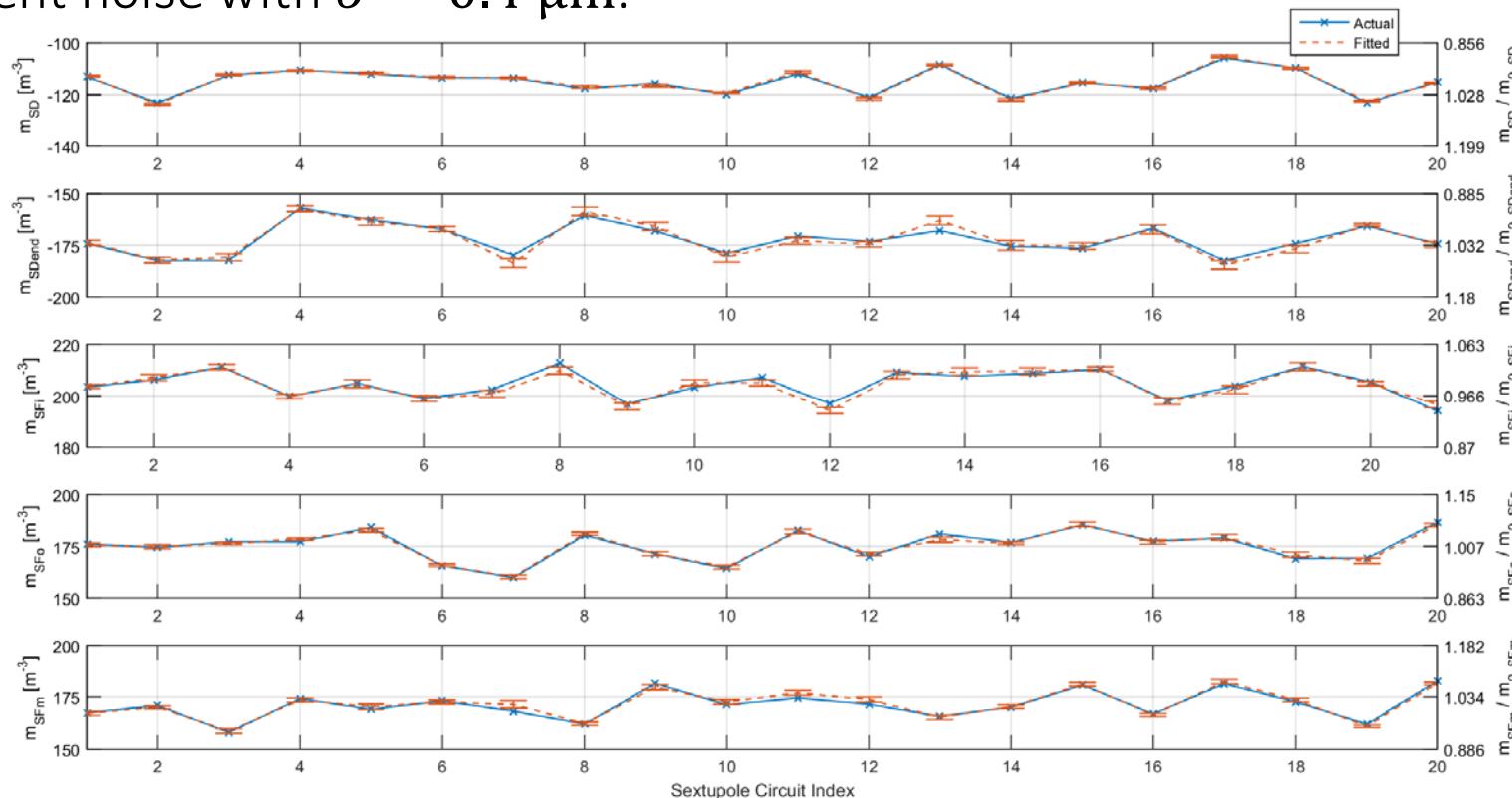
$$\eta = \eta_1 + \eta_2 \delta + \eta_3 \delta^2 + \dots$$

- The 2nd order dispersion η_2 is linearly dependent on BPM gains and chromatic sextupoles, and independent of dipole correctors.



Simulation: Characterisation of Model

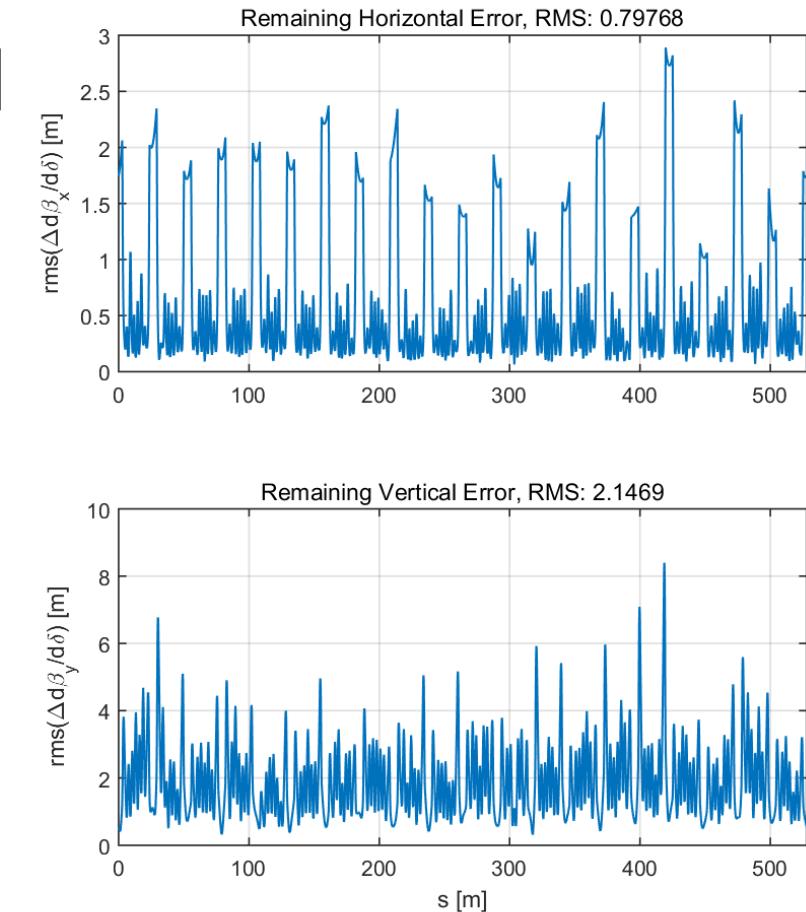
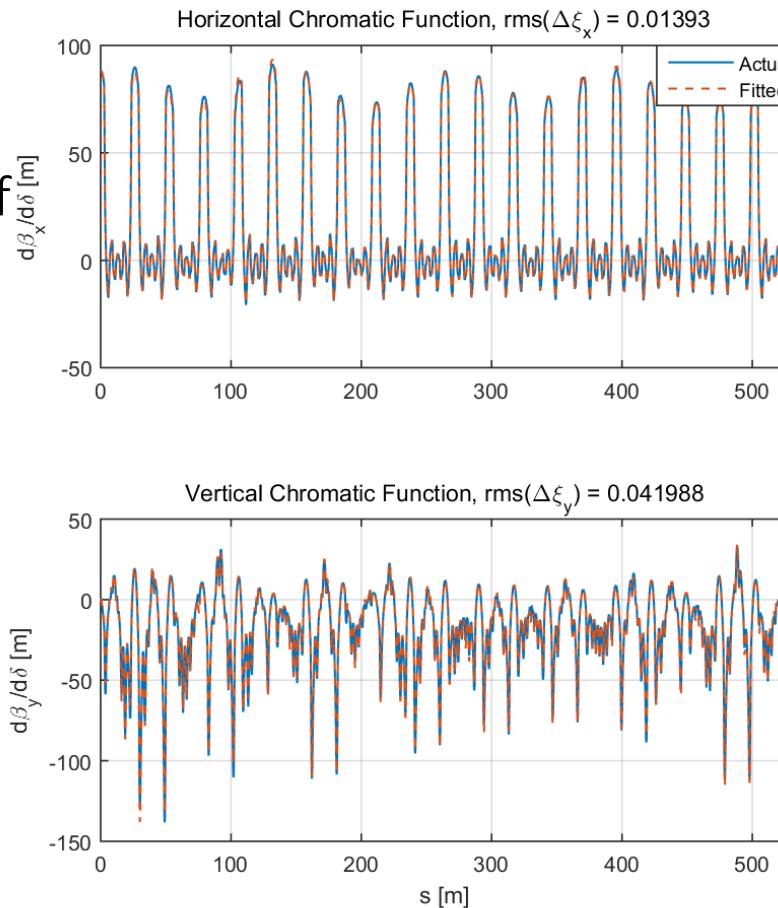
- Sextupole **circuit** errors with $\sigma = 4\%$, cut-off at 2σ .
- Alignment errors with $\sigma = 25\text{ }\mu\text{m}$, cut-off at 2σ .
- Measurement noise with $\sigma = 0.4\text{ }\mu\text{m}$.



Actual and identified sextupole circuit strengths.

Simulation: Characterisation of Model

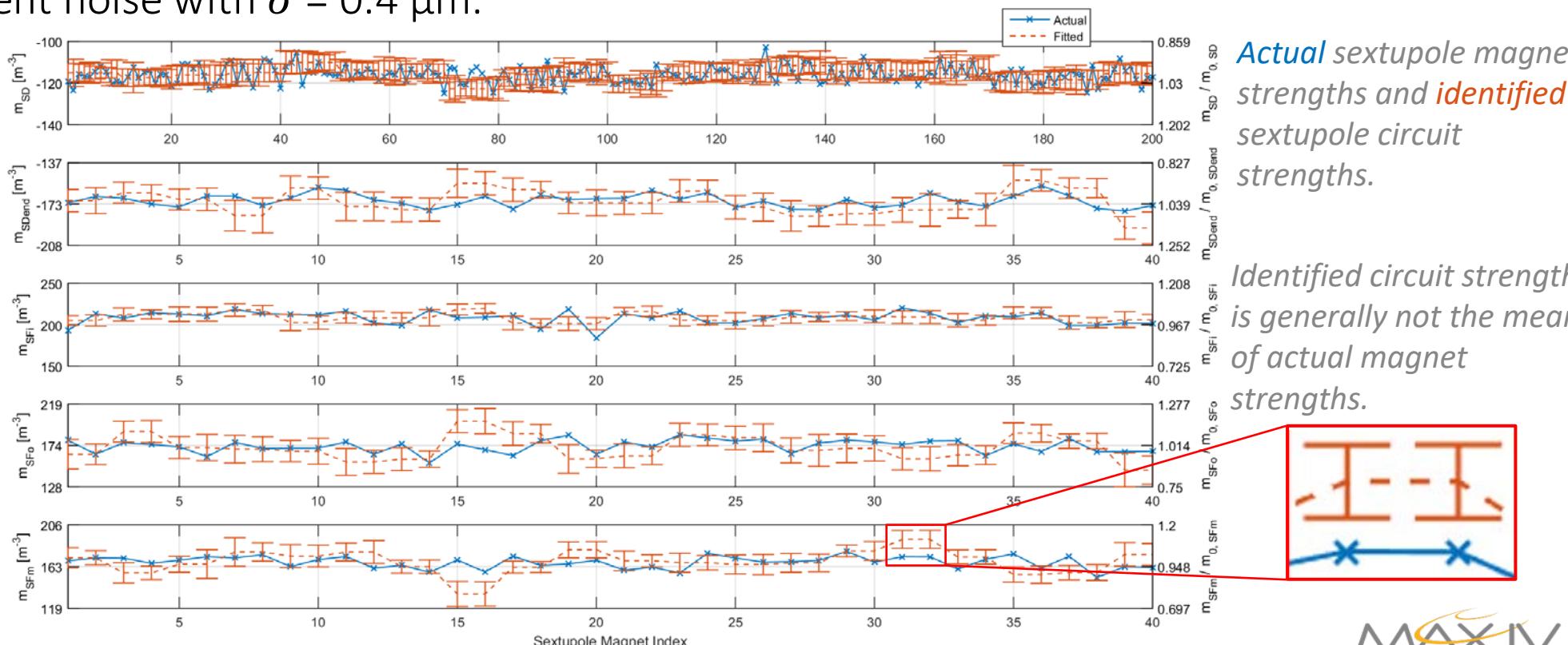
- Using the chromaticity and the chromatic functions as a measure of fit accuracy.
- The chromatic functions of the model with circuit errors is well reproduced.



Left: Chromatic functions of model with circuit errors and the NOECO fitted model.
Right: RMS error of fits to 10 error seeds.

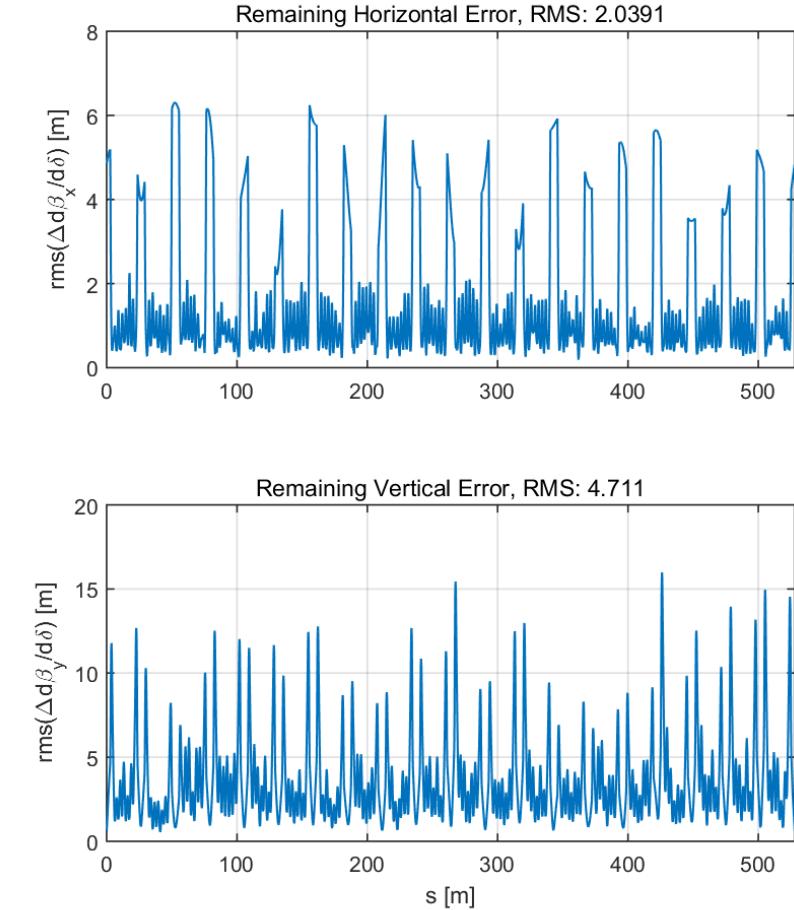
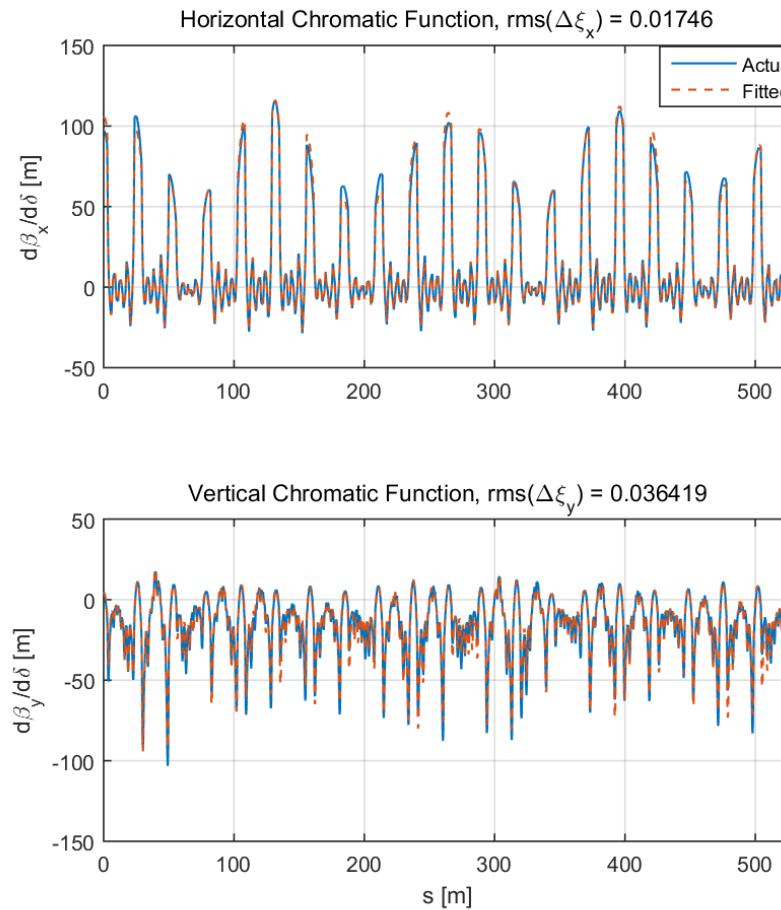
Simulation: Characterisation of Model

- Sextupole **magnet** errors with $\sigma = 4\%$, cut-off at 2σ .
- Alignment errors with $\sigma = 25\text{ }\mu\text{m}$, cut-off at 2σ .
- Measurement noise with $\sigma = 0.4\text{ }\mu\text{m}$.



Simulation: Characterisation of Model

- The chromatic functions of the model with magnet errors is also well reproduced, despite the errors being outside the parameter space of the fit.

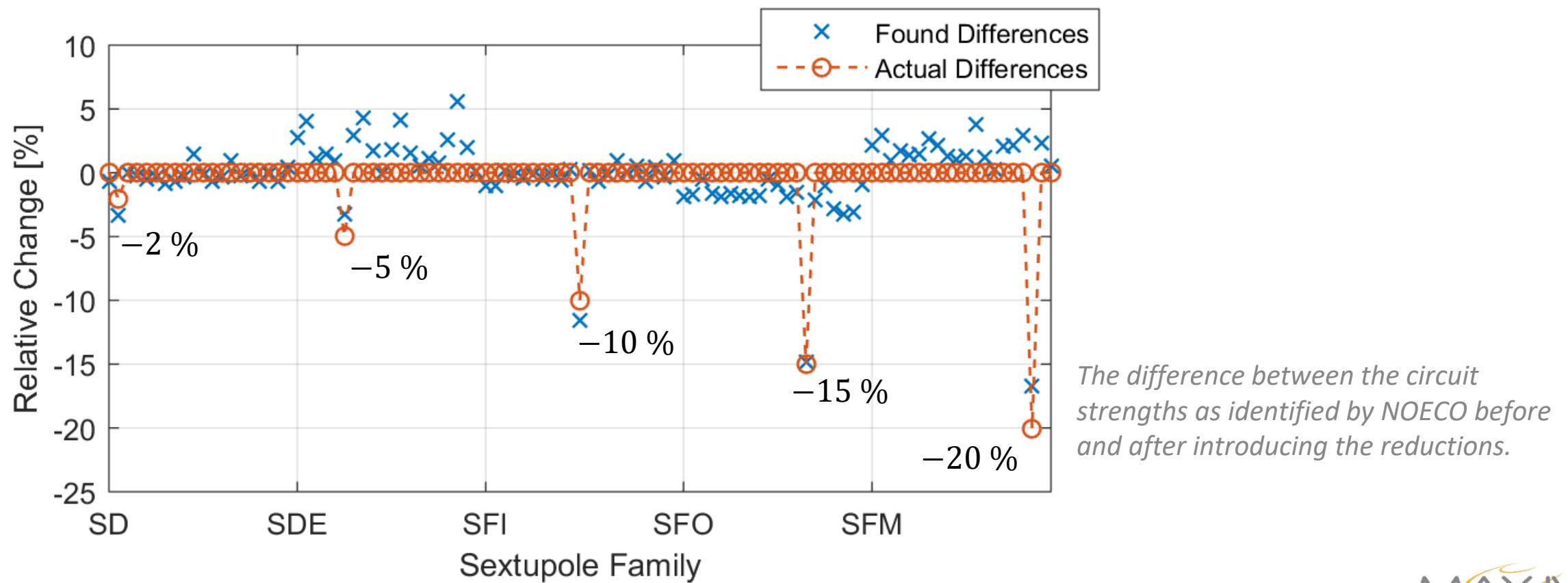


*Left: Chromatic functions of model with magnet errors and the NOECO fitted model.
Right: RMS error of fits to 10 error seeds.*

Proof-of-Concept Measurements

Detection of reductions of sextupole circuit strengths:

-2% SD, -5% SDE, -10% SFI, -15% SFO, -20% SFM



Proof-of-Concept Measurements

Detection of reductions of sextupole circuit strengths:

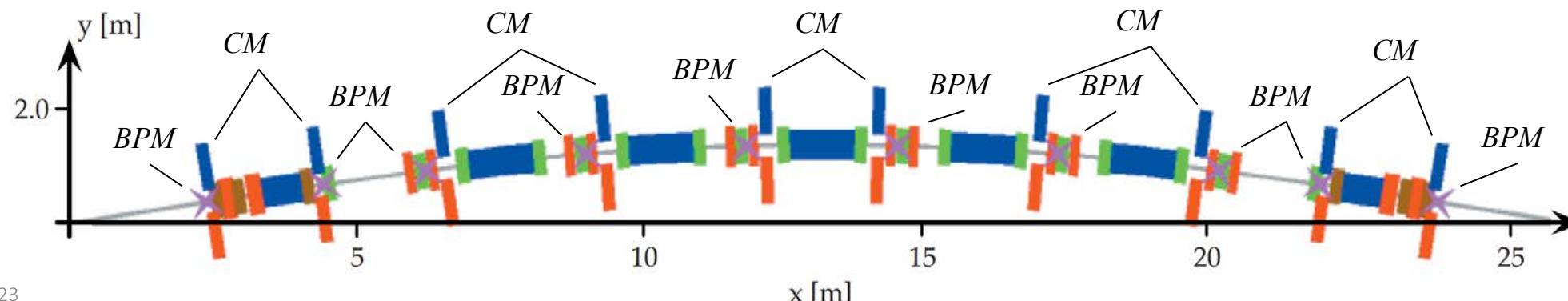
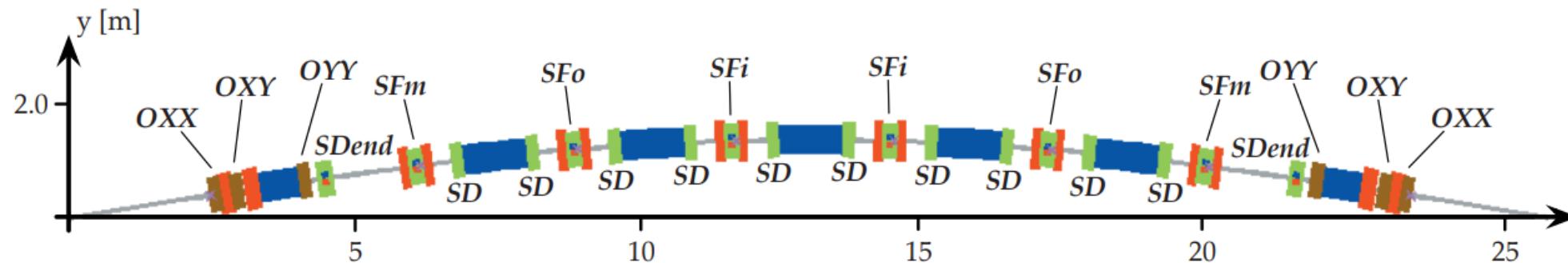
-2% SD, -5% SDE, -10% SFI, -15% SFO, -20% SFM

	Measured ξ_x/ξ_y	Fitted ξ_x/ξ_y
Before errors	+1.1200/ + 0.8531	+1.0169/ + 0.9889
After errors	+0.5342/ + 1.3590	+0.5047/ + 1.6117

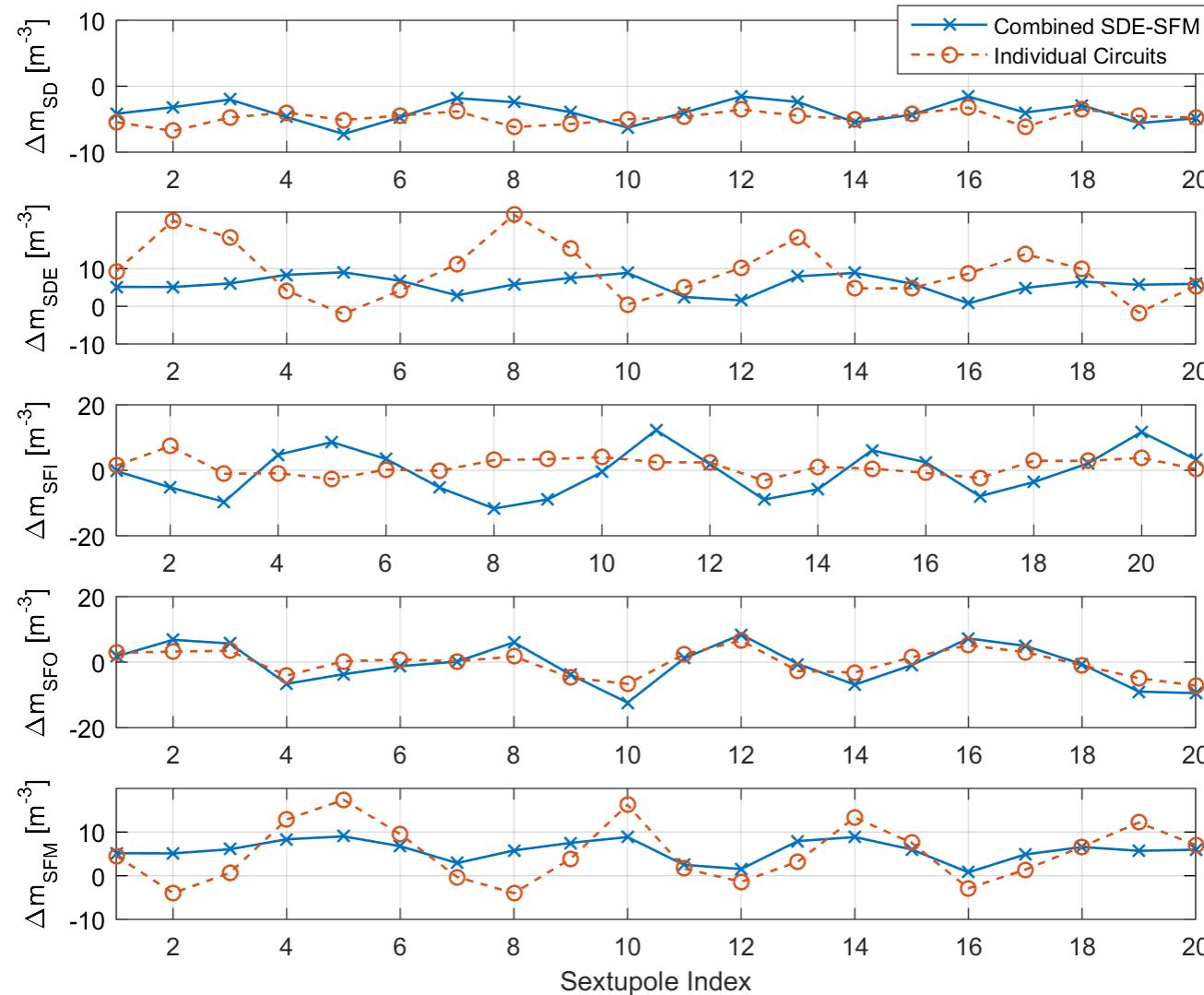
Measured chromaticity and chromaticity from the NOECO fit before and after applying the intentional sextupole errors.

Singularity Issues

- Initial correction saturated several SDend circuits.
- No BPMs or dipole correctors between the SDend and SFm sextupole magnets.
- No measurement points -> singularity in the fitting procedure.



Singularity Issues

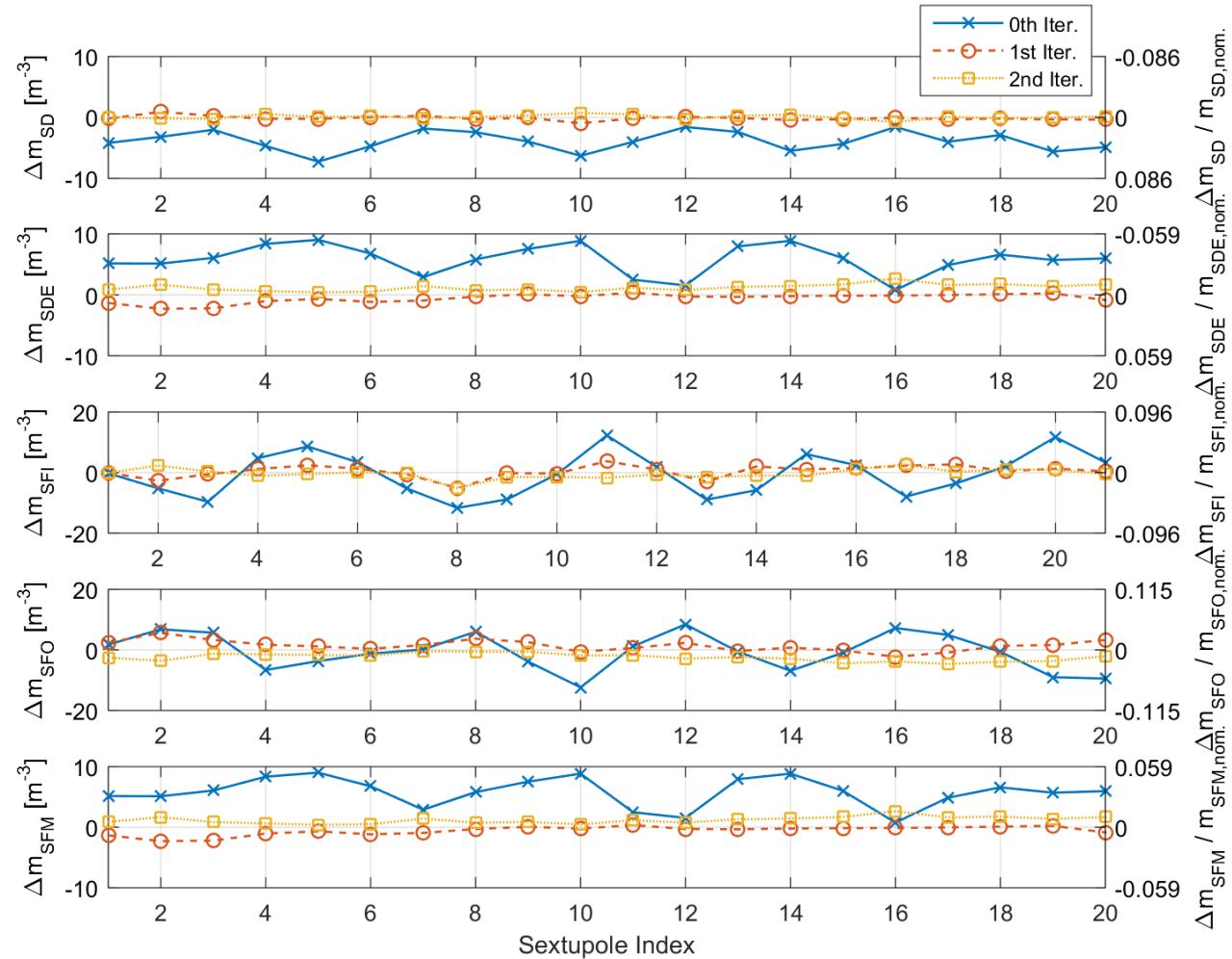


Proposed sextupole corrections when *fitting each circuit individually* or when *combining the neighbouring SDE and SFM circuits*. Combining the circuits resulted in much smaller corrections to the SDE and SFM families.

Iterative Correction of 2nd Order Optics

- Corrections applied iteratively.
- Sextupole errors from non-sextupole magnets.
 - Sextupole fields in other magnets.
 - Alignment errors.
 - Etc.

Proposed sextupole corrections after each iteration of corrections were applied to the machine. As the sextupole settings converge, the correction become smaller.

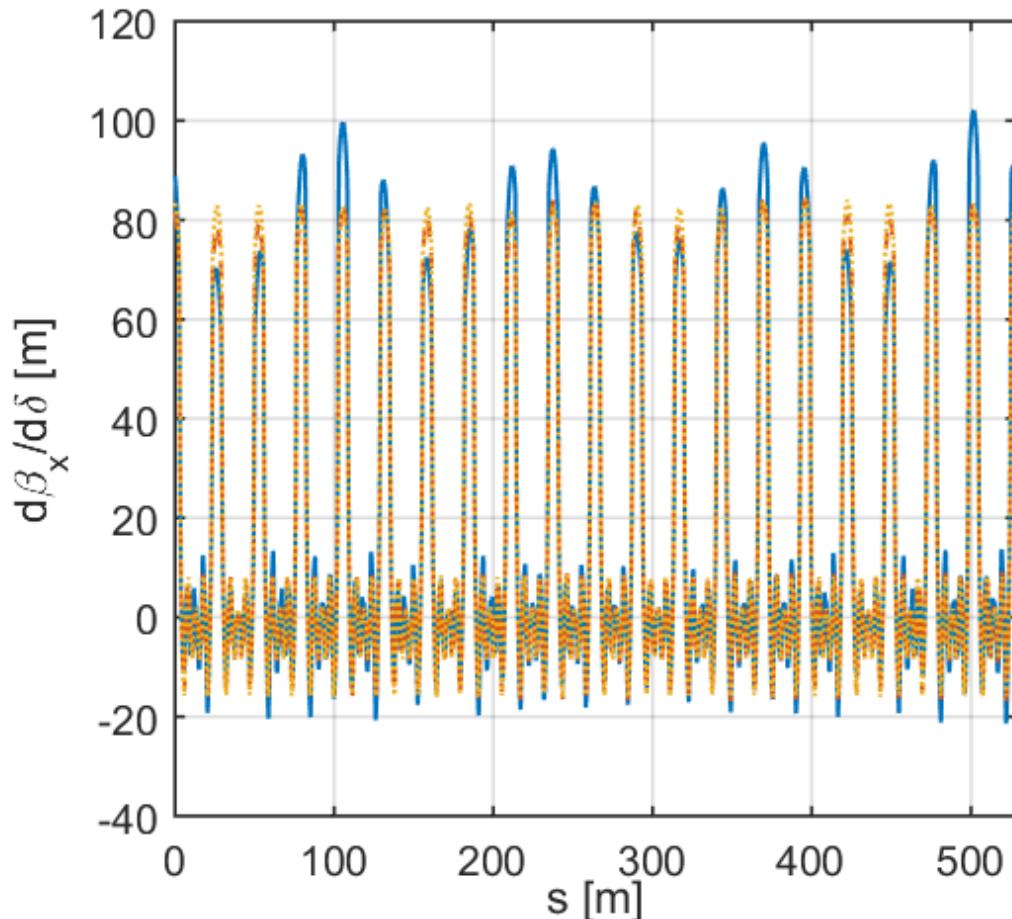


Chromaticity Convergence

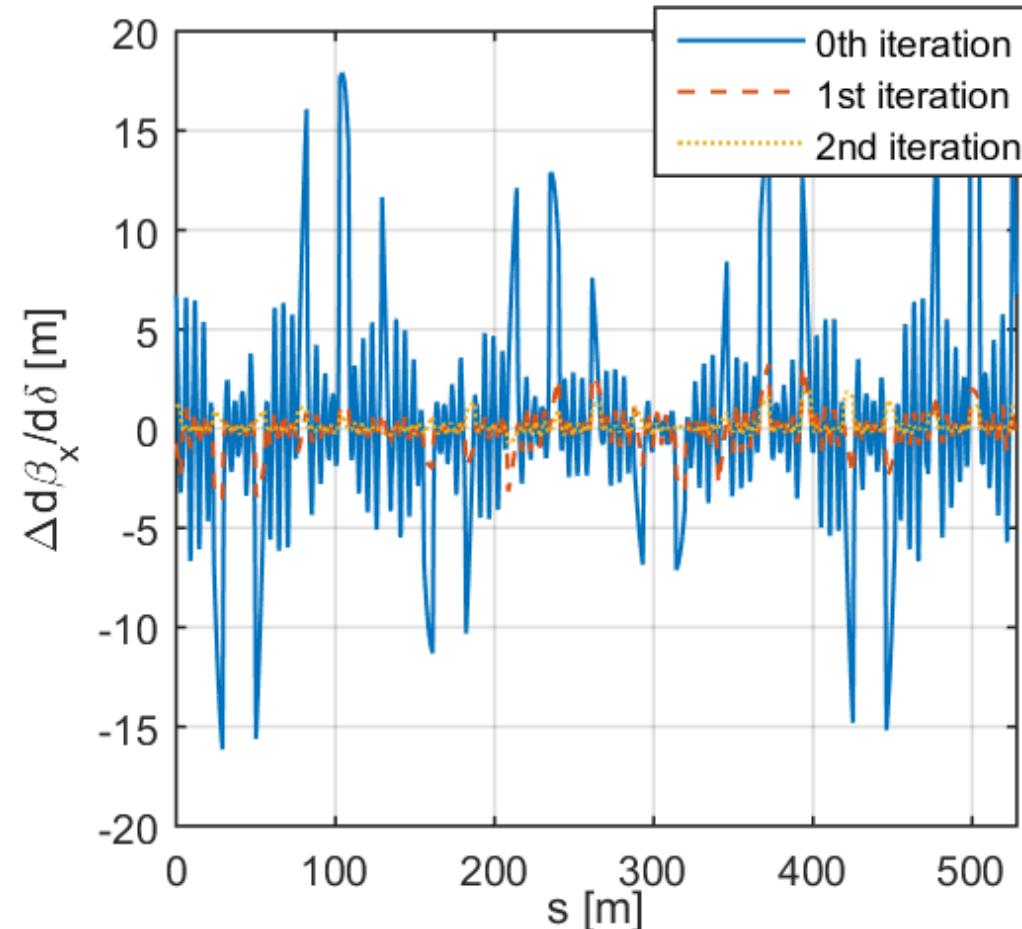
	Measured ξ_x/ξ_y	Fitted ξ_x/ξ_y
0th iteration	+0.9233/ + 3.2345	+0.7873/ + 3.2507
1st iteration	+1.2167/ + 0.8254	+1.1884/ + 0.9677
2nd iteration	+1.0089/ + 0.9722	+0.9963/ + 0.9948

Measured chromaticity and chromaticity from the NOECO fit with each iteration of applying correction to the sextupole circuits.

Chromatic Function Convergence

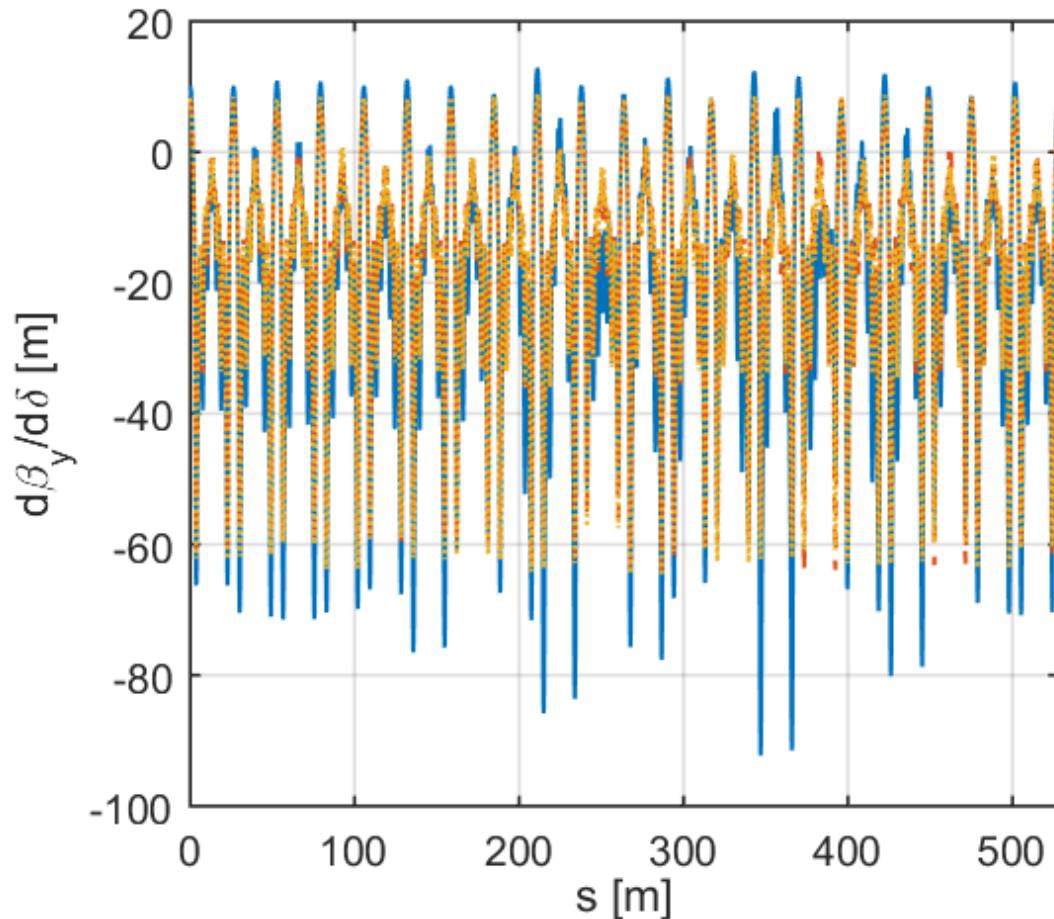


Horizontal chromatic functions with each iteration of the NOECO procedure.

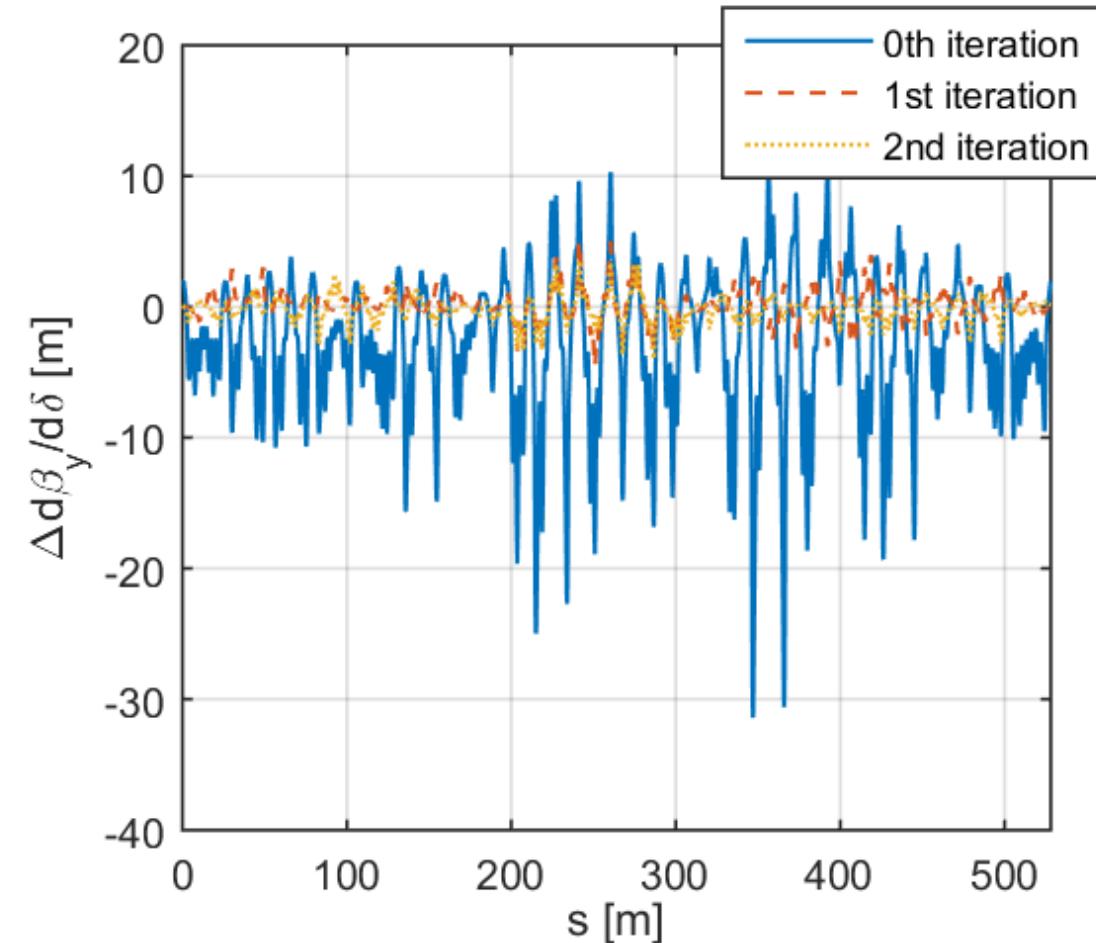


Horizontal chromatic function error with each iteration of the NOECO procedure.

Chromatic Function Convergence

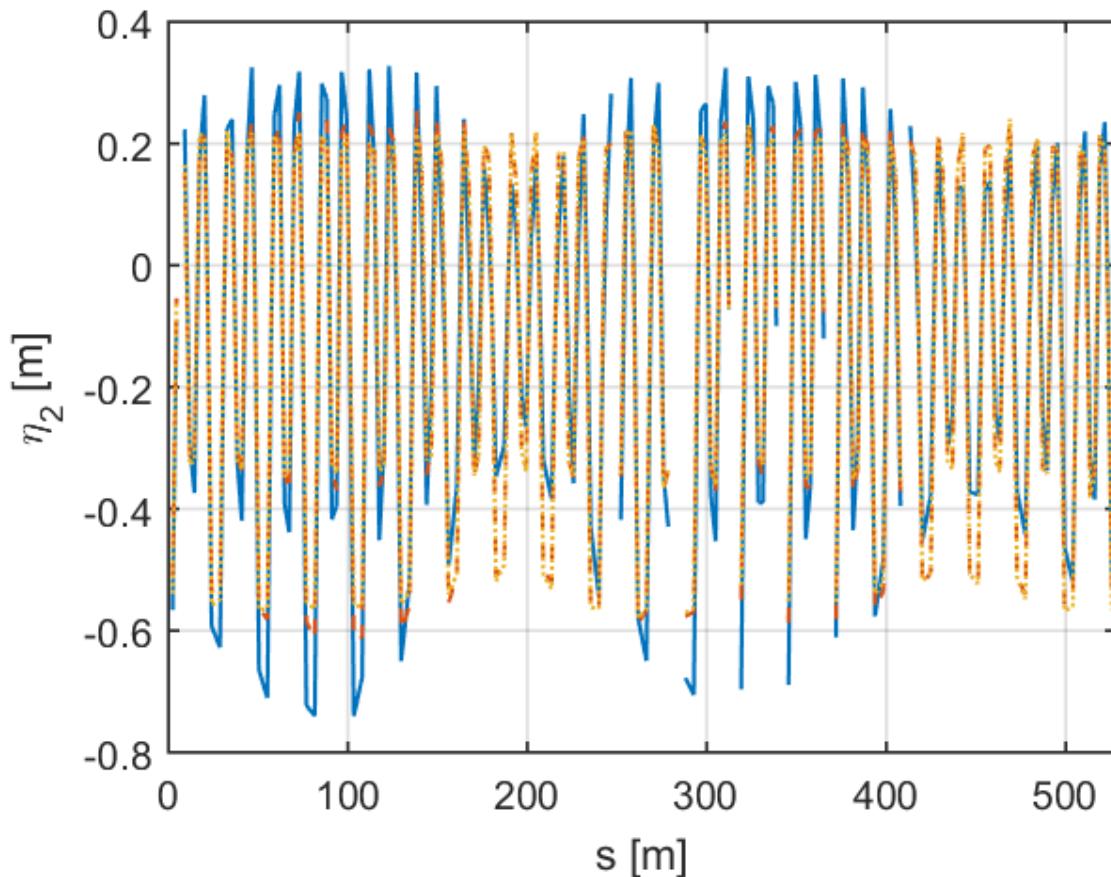


Vertical chromatic functions with each iteration of the NOECO procedure.

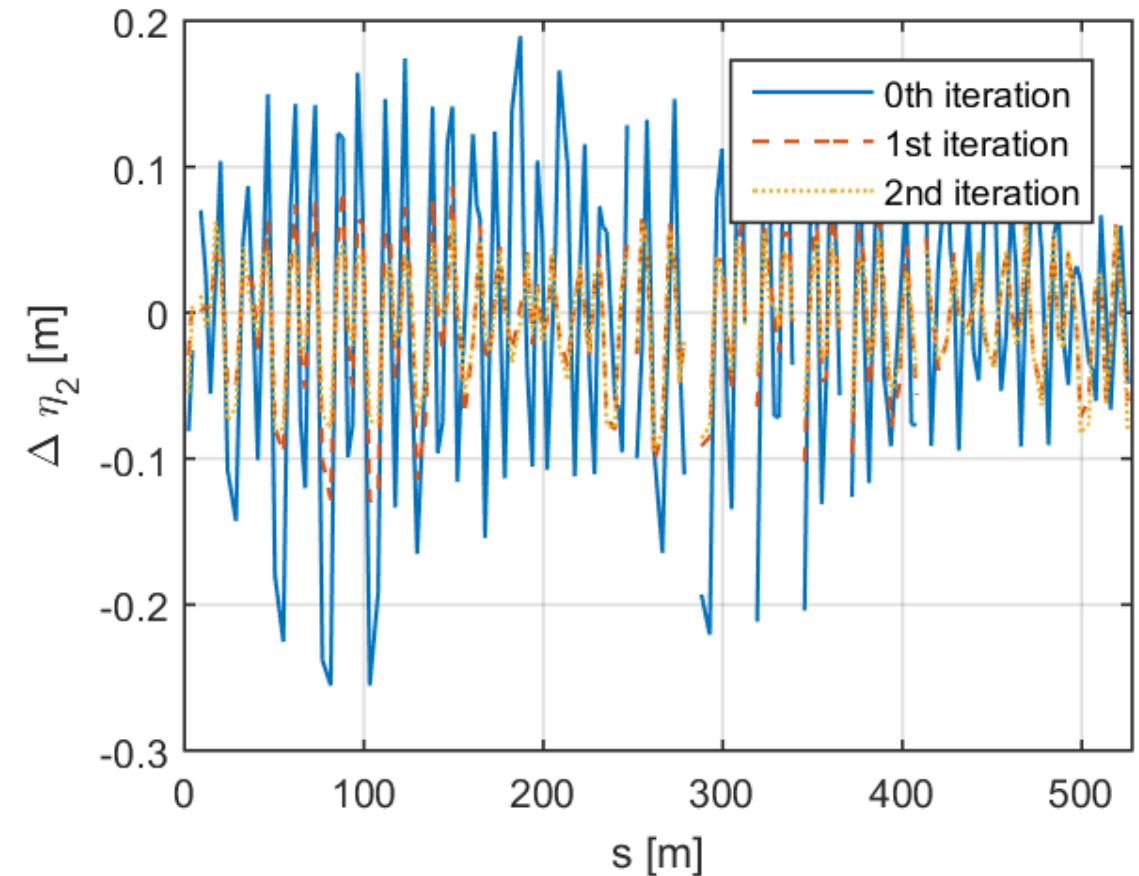


Vertical chromatic function error with each iteration of the NOECO procedure.

2nd Order Dispersion Convergence

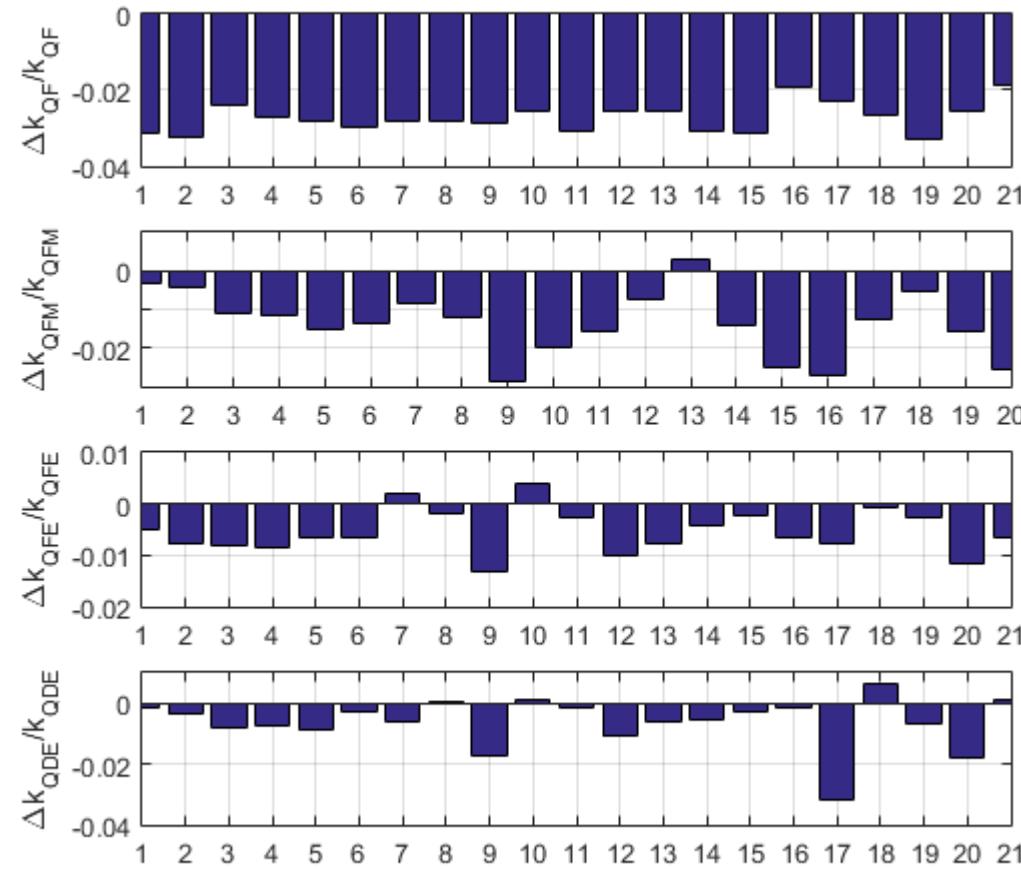


Horizontal 2nd order dispersion with each iteration of the NOECO procedure.

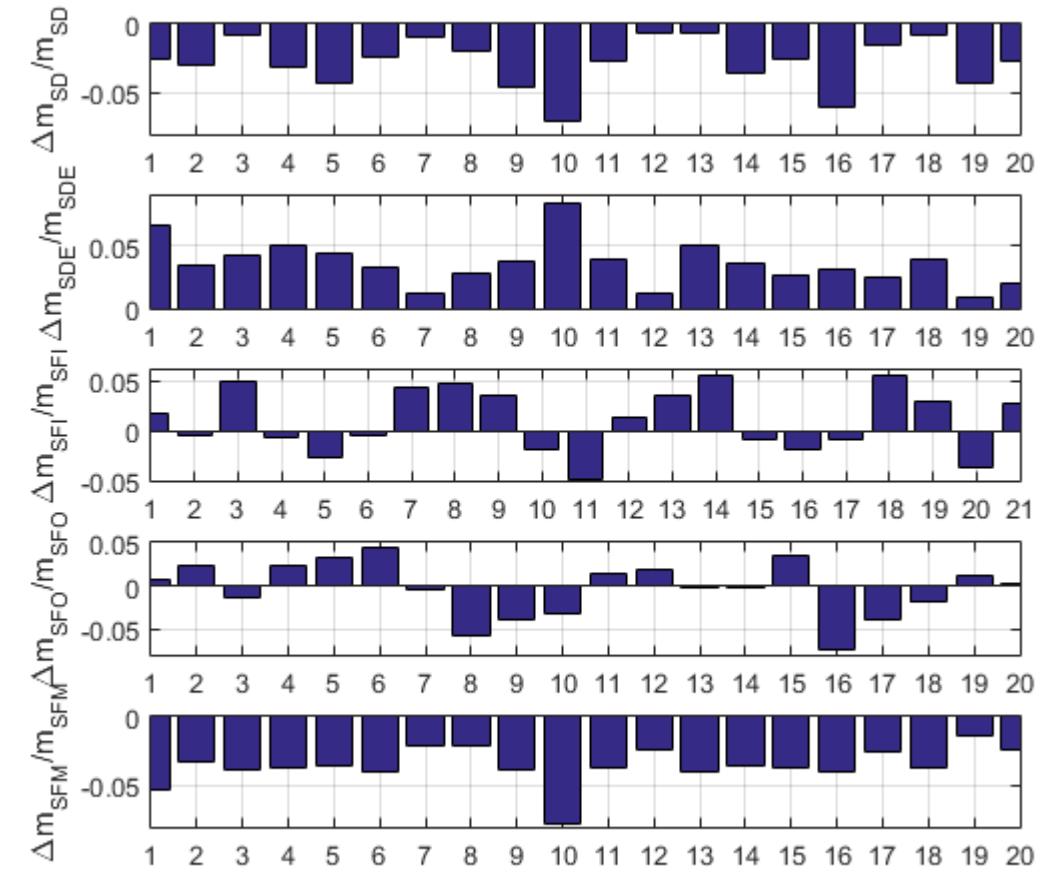


Horizontal 2nd order dispersion error with each iteration of the NOECO procedure.

Total Change of Magnet Strengths

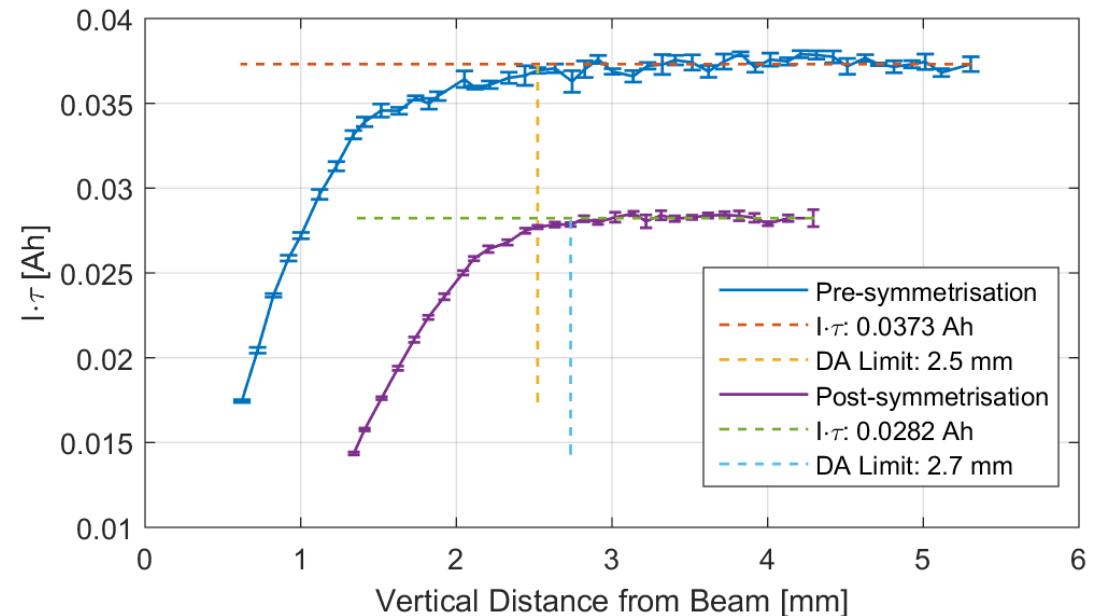
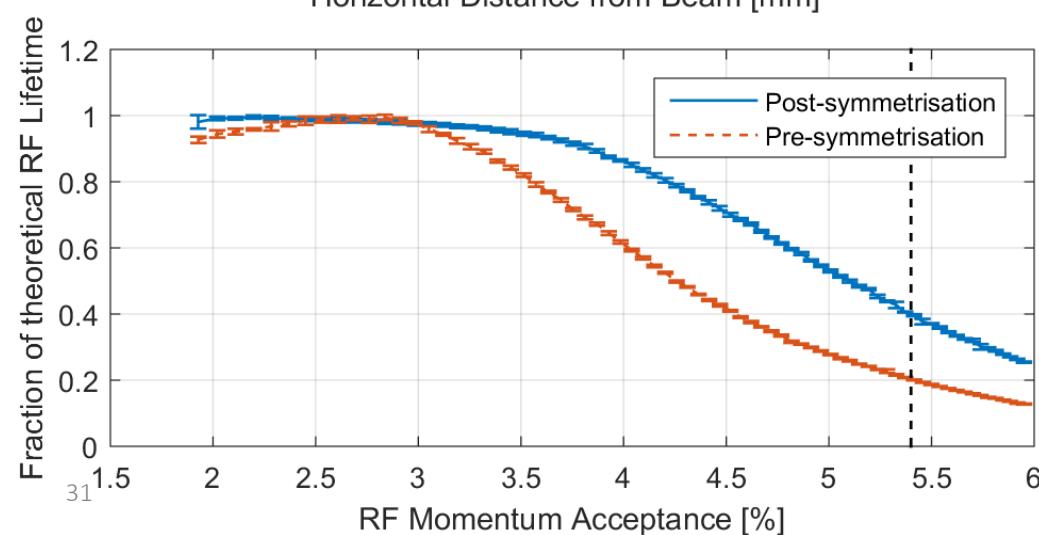
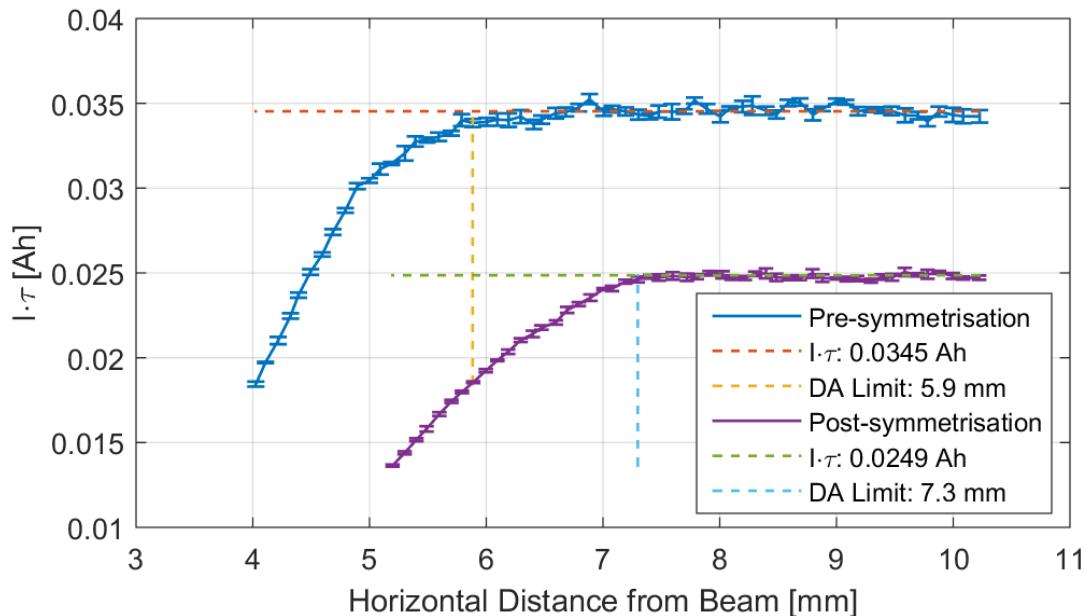


Relative change of magnet settings arrived at by iterative applications of LOCO.



Relative change of magnet settings arrived at by iterative applications of NOECO.

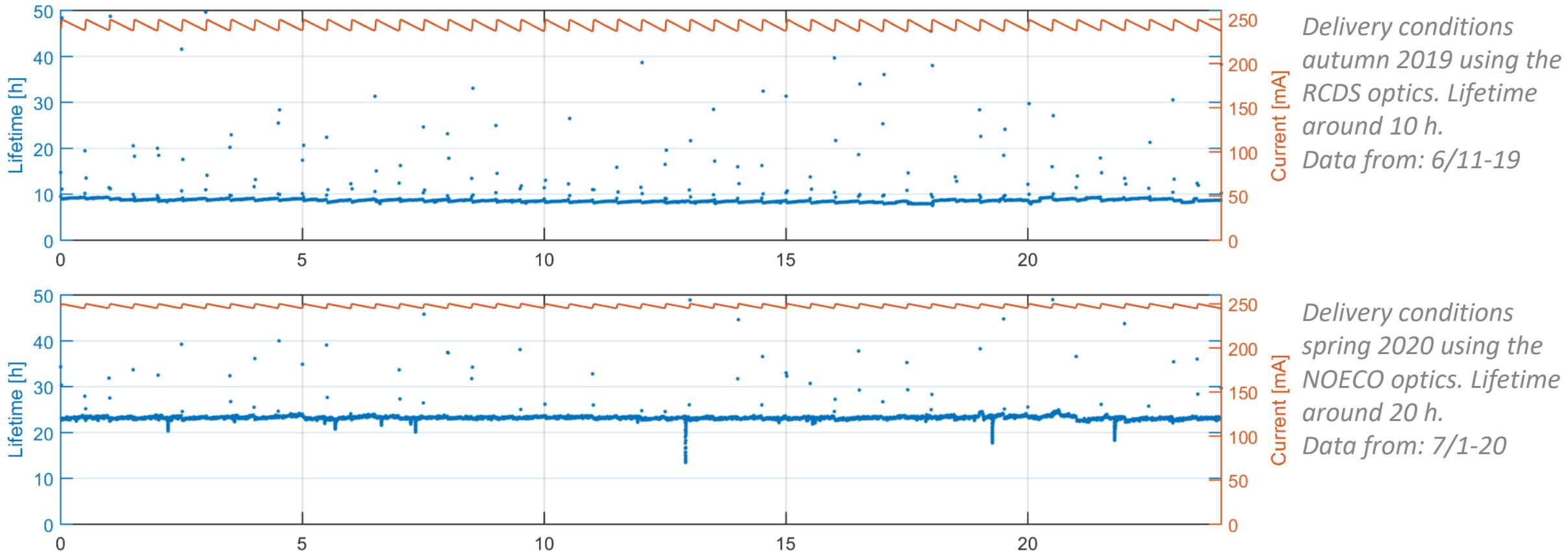
Effect on Dynamic Aperture



*Top: Dynamic aperture as measured by scrapers.
The pre-symmetrised linear optics had a larger
emittance coupling.*

*Bottom: Measured Touschek lifetime relative to
theoretical value.*

Effect on Dynamic Aperture



Summary

Sextupole settings of the MAX IV 3 GeV storage ring were found using the NOECO scheme.

- Characterisation and correction towards model values.
- Peak sextupole errors 3x larger than quadrupole errors.
- Machine performance approaches model.
- Can be applied to most lattices.
- Can only characterise chromatic sextupoles.

Thank You

David K. Olsson, Åke Andersson, and Magnus Sjöström.
“Nonlinear optics from off-energy closed orbits”, *Phys. Rev. Accel. Beams* 23, 102803 (2020).
<https://doi.org/10.1103/PhysRevAccelBeams.23.102803>

References

- [1] J. Safranek. “Experimental determination of storage ring optics using orbit response measurements”, *Nucl. Instrum. Meth. A* 388, pp. 27-26 (1997).
DOI: 10.1016/S0168-9002(97)00309-4
- [2] P. F. Tavares et al. “Commissioning and first-year operational results of the MAX IV 3 GeV ring”, *J. Synchrotron Radiat.* 25(Pt 5), pp 1291-1316 (2018).
DOI: 10.1107/S1600577518008111

- [3] X. Huang, J. Corbett, J. Safranek, J. Wu. “An Algorithm for Online Optimization of Accelerators”, *Nucl. Instrum. Meth. A* 726, pp. 77-83 (2013).
DOI: 10.1016/j.nima.2013.05.046
- [4] D. K. Olsson. “Online Optimisation of the MAX IV 3 GeV Ring Dynamic Aperture”, *Proc. of IPAC18*, Vancouver, Canada, WEPAL047, pp. 2281-2283 (2018).
DOI: 10.18429/JACoW-IPAC2018-WEPAL047